MBS Outstanding Thesis

Master Thesis

Bottom-up Portfolio Construction of Public Return Factors: Application to European Private Equity Fund Benchmarking

Munich Business School Working Paper

2020-03

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Munich Business School Working Paper Series, ISSN 2367-3869

This master thesis was submitted to Munich Business School in August 2020 and supervised by MBS Prof. Dr. Eva Stumpfegger (Munich Business School) and Mister Christian Tausch (Firma AssetMetrix GmbH).

Abstract

This study aims to calculate returns on the risk factors of the European stock market with a newly popularized asset pricing model and apply these factor returns to benchmark the performance of the European private equity (PE) funds. Adopting the q-factor model of Hou et al. (2015), this thesis explains the market excess return in Europe with four independent variables: a market factor, a size factor, an investment factor, and a profitability factor. Based on a broad European stock sample consisting of 5777 companies over the period from January 1981 to December 2019, the returns on these q-factors are computed with a bottom-up portfolio construction approach. This study examines the validity of the q-factor pricing model with a cross-sectional regression methodology and demonstrates that this model is well specified in the European market. By embedding the q-factor returns into the net present value (NPV) based framework of Driessen et al. (2012), the systematic risk and abnormal returns of the European PE funds are estimated and the performance of both buyout segment and venture capital segment is analyzed. In this manner, the study identifies a significant negative size factor premium of -0.65% per month on average. Surprisingly, the average monthly investment factor return is also negative (-0.41%). The market factor earns an average monthly premium of 0.69% and the profitability factor generates a monthly return of 0.79%, moderately higher than that of the U.S. equity market. The empirical results of European PE fund performance - with regards to the alpha and beta estimations - when benchmarked with the q-factor returns, are broadly consistent with existing literature. This research estimates a positive abnormal return of 4.1% per annum for buyout segment and a negative abnormal return of -5.6% for venture capital. The overall market beta of buyout funds is slightly bigger than 1 and for venture investment, it ranges from 0.97 to 2.33. Venture capital funds are exposed to higher market risk but receive less return than the buyout funds. Another finding of this study is that the inclusion of the q-factors in the capital asset pricing model (CAPM) does reduce the pricing error of private equity fund performance evaluation. The model that contains the alpha and the market factor is the best specification for European buyout funds. For venture capital funds in Europe, the model which incorporates the profitability factor and the market factor best specifies the risk and return.

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Index of Abbreviations

CA	Cambridge Association
CAPM	Capital Asset Pricing Model
CMA	Conservative-minus-Aggressive
DPI	Distribution to Paid-in Capital
EONIA	Euro Overnight Index Average
Eg	Expected Growth Factor
FIN	Financing Factor
FoF	Fund-of-Fund
GMM	Generalized Method of Moments
GP	General Partner
GPME	Generalized Public Market Equivalent
CRSP	Center for Research in Security Prices
HML	High-minus-Low
I/A	Investment-to-Assets
IRR	Internal Rate of Return
KS-PME	Kaplan and Schoar Public Market Equivalent
LHS	Left-Hand-Side
LP	Limited Partner
ME	Market Equity
MGMT	Management Factor
MKT	Market
MOM/UMD	Momentum Factor
NAV	Net Asset Value
NI/CFO	Net Income/Cash Flow Generated from Operating Activities
NPV	Net Present Value
NYSE	New York Security Exchange
PE	Private Equity
PEAD	Post-Earnings Announcement Drift Factor
PERF	Performance Factor
PME	Public Market Equivalent
PMU	Profitable-minus-Unprofitable
PV	Present Value
RHS	Right-Hand-Side
RMW	Robust-minus-Weak

ROE	Return on Equity
SDF	Stochastic Discount Factor
SMB	Small-minus-Big
SML	Security Market Line
T-bill	Treasury Bill
TVE	Thomson Venture Economics
TVPI	Total Value to Paid-in Capital

1. Introduction

Asset pricing has been a prominent research area in the field of finance since Sharpe introduced the capital asset pricing model (CAPM) in 1964, which describes a linear relationship between return and systematic risk. A number of studies have been conducted to find return anomalies, to identify risk factors, to measure explanatory variables, and to build, examine and compare different augmented asset pricing models over the past decades (e.g., Carhart, 1997; Fama and French, 1993, 2015, 2018; Hou, Xue, and Zhang, 2015, 2017; Stambaugh and Yuan, 2017; Daniel, Hirshleifer and Sun, 2020; Hou, Xue, Mo, and Zhang, 2019, 2020). Asset pricing research can be used to determine the capital cost or the price of an asset, to predict expected return of investment, to explain returns attributed to different risk variables, and to evaluate portfolio or fund performance (Walkshäusl and Lobe, 2014). The widely acknowledged Fama-French factor models (1993, 2015) remain the most popular among all the asset pricing models; that is, until the publication of Hou et al. (2015, 2017, 2019, 2020) which steal the spotlight.

Hou et al. (2015) propose a q-factor model that captures return patterns related to four factors: market (MKT), size (ME), investment (I/A), and profitability (ROE). And in 2020, they add an expected growth factor (Eg) to create the q^5 factor model. By stress-testing on a wide range of anomalies, they prove that the q-factor model significantly subsumes the Fama-French models in capturing return anomalies. Furthermore, the q^5 factor model has the strongest explanatory power of the excess return among all competing models. Since these publications constitute recent additions to the literature, very few attempts have been made to validate these models empirically, particularly in non-U.S. markets. For the time being, the q-factor returns are available for the U.S. equity market[1]. No previous study has worked on the q-factor returns of the European stock market except for a new paper of Huber and Preissler (2020). This research only studies a small sample over the period from 1990 to 2018, and the main objective is to compare influential pricing models from behavioral finance and neoclassical finance. On the other hand, literature related to the application of q-factor models is rather limited, especially towards the portfolio performance evaluation.

[1] The q-factor returns are computed by Hou, Xue, and Zhang and released on their official website: http://global-q.org/factors.html

Currently, neither of these two advanced models has been applied to analyze the performance of private equity investment.

Given the aforementioned research gaps, this thesis aims to calculate the q-factor returns in Europe more accurately - based on a much broader and more comprehensive dataset over a longer period, and to apply these factor returns to benchmark European PE fund performance with a bottom-up solution. This thesis seeks to address the following research questions: 1) How do the factors of the q-factor model explain European public equity market returns? 2) How can these q-factor returns be applied for the benchmarking of European private equity fund performance? Considering the large and persistent home (US) bias observed in finance academic research (Karolyi, 2016), this study focuses on the European market[2] by using European public and private equity datasets. The q-factor model, which contains four factors, is selected to perform this study concerning the high difficulty level of the expected growth factor computation in the q^5 factor model.

For the first research question, this thesis collects listed companies from the benchmark public equity indices to form an integrated European index as the study sample. After a comprehensive data screening process on the time-series security accounting data, this study follows the portfolio construction procedure outlined by Hou et al. (2015) to replicate the q-factors for the European equity market. This thesis explains the European stock returns in excess to the risk-free rate with the computed four q-factors. The validity of the q-factor pricing model is tested with a cross-sectional regression methodology. The empirical result shows that this model is well specified in the European market. To answer the second research question, the q-factor returns and real cash flow streams of European PE funds are used as input in a modified NPV-based model (Driessen, Lin, and Phalippou, 2012). With this estimator framework, the average systematic risk and abnormal returns of the private equity fund[3] over the period of 1985 to 2019 are estimated.

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[2] In this paper, the European market comprises 34 countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Liechtenstein, Lithuania, Luxembourg, Malta, Monaco, Netherlands, Norway, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, and United Kingdom.

[3] Unless otherwise specified, private equity fund in this paper refers to buyout fund and venture capital.

The first contribution of this thesis is to fill the gap in the existing literature by constructing and calculating the q-factor returns at a European level. This is one of the first studies to employ the q-factor model to explain the cross-sectional average return in Europe. These q-factors can be useful in measuring fund performance, estimating capital cost, determining discount rate for the valuation of cash flows, and providing insights for decision making on European investments.

Another contribution is that this study serves as a pioneer in evaluating the European private equity fund performance with the q-factor returns as benchmarks. Despite the rapid development of the private equity industry, it remains a complicated task to measure the return and performance for private equity investment due to its illiquid nature and lack of transparent information (Ang, Chen, Goetzmann, and Phalippou 2018). The Driessen NPV-based framework is one of the most comprehensive methods to estimate the risk and return of PE assets, which considers both the time value of money and flexible risk factors. Initially, the CAPM and the Fama-French 3-factor model (1993) are used for risk and return estimation. In the light of Tausch (2020), this thesis enhances the Driessen approach by employing an average NPV and incorporates the q-factor asset pricing model with the modified NPV framework.

This thesis is structured as follows: Chapter 2 discusses related literature. Chapter 3 describes the q-factor model applied for factor return calculation and the methodology of private equity fund performance analysis. Chapter 4 elaborates on the data process and empirical results of q-factor returns in Europe. Chapter 5 presents the application of the q-factor returns in the benchmarking of private equity funds.

2. Literature Review

2.1 The Theoretical Framework of Asset Pricing

The stochastic discount factor (SDF) is the fundamental of asset pricing. It implies a valuation theory that pricing security or portfolio can be achieved by computing the present value (PV) of the future cash flows discounted for risk and time lags as follows:

$$PV = E\left[\sum_{t=0}^{T} \frac{X_t}{\prod_{s=0}^{t}(1+R_s)}\right] \tag{1}$$

where X_t is the cash flow at period t and $\frac{1}{\prod_{s=0}^{t}(1+R_s)}$ represents the stochastic discounting factor for the cash flows. Celik (2012) points out that the difficulty of this process is to determine the relevant risk factors which affect the cash flows.

In light of the earlier studies in terms of diversification and modern portfolio theory (Markowitz, 1959), Sharpe (1964) introduces the CAPM which decomposes the return into adjusted market factor and a risk-free rate. As a fundamental asset pricing model, it still remains popular due to its simplicity. The CAPM is presented as:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{2}$$

where $E(R_i)$ is the expected return on the capital asset i, R_f is the risk-free return which accounts for the time value of money, R_m is the expected market return and β_i is denoted as the systematic risk - which reflects the sensitivity of the expected excess asset returns to the market premium. This can be regarded as the security market line (SML), where the slope of the SML is interpreted as the risk coefficient. The CAPM implies a linear relationship between the risk factors and the excess return. Assuming there are multiple factors which can predict the return, let the number of factors be K, and for factor k (k=1, …, K), the respective risk premium is λ_k. Therefore, a multi-factor model can be structured as:

$$E(R_i) - R_f = \sum_{k=1}^{K} \beta_{ik}\lambda_k \tag{3}$$

where $E(R_i) - R_f$ is the expected excess return of asset i, β_{ik} is the risk loading on factor k. For existing portfolios with realized returns, equation 3 can be tested by the following time-series regression (Flechter, 2019):

$$r_{it} = \alpha_i + \sum_{k=1}^{K} \beta_{ik} r_{kt} + \varepsilon_{it} \tag{4}$$

where r_{it} denotes the excess return of asset i at time t, r_{kt} denotes the average excess return resulting from factor k at time t, α_i is the pricing error of asset i, ε_{it} denotes the random error term. For a factor model to be well specified, the absolute value of α_i should be insignificant and close to 0, because α_i stands for the return that cannot be explained by this factor model. The logic can also be applied to compare different asset pricing models. The one with the smallest intercept value performs best.

2.2 An Overview of Multi-Factor Pricing Models

The CAPM can be viewed as a one-factor model. It states that the expected return of an asset is equal to the risk-free rate plus risk premium from the market factor. This conventional model, in that it only contains one variable to describe the returns of securities, has proven insufficient, but it sheds light and shapes the way for further asset pricing related research.

Banz (1981) discovers the size effect by noticing a relatively high average return on stocks with low market capitalization (i.e., small stocks). Fama and French (1993) find value stocks (i.e., companies with a low book-to-market ratio) tend to perform better than the market. They test both size and value factors on the return of NYSE, Amex, and NASDAQ stocks for the period 1963-1990 and extend the CAPM with two variables - SMB and HML - to form the Fama-French 3-factor model. SMB is the return on a diversified portfolio of small-cap stocks minus the return on a diversified portfolio of big-cap stocks, HML is the difference between the returns on high and low B/M stock portfolios. Fama and French (1993) suggest that the stock risks are multidimensional, a concept which raises a surge of research on different factors that can explain anomalies in cross-sectional asset pricing (Harvey, Liu, and Zhu 2016). Interestingly, instead of identifying a large number of predictors like in earlier research, current literature focuses on discovering and testing the most relevant factors. This major shift helps to avoid data mining issues in finance research as well as unnecessary efforts to analyze and testify anomalies which could be incorrect or irrelevant (Huber and Preissler, 2020).

Carhart (1997) initiates a momentum factor (MOM) and comes up with a 4-factor model which is an extension of the Fama-French 3-factor model. The work of Pástor and Stambaugh (2003) finds out that the fluctuated aggregate liquidity is highly related to cross-sectionally expected stock returns. Additionally, the empirical result reports an annual difference of 7.5% between the average return on stocks with high sensitivities to liquidity and the average return on stocks with low sensitivities. Chen, Novy-Marx, and Zhang (2011) propose a novel 3-factor model that contains an earnings-to-assets factor (PMU, Profitable-minus-Unprofitable). Foye, Mramor, and Pahor (2013) find the NI/CFO (net income/cash flow generated from operating activities) has better explanatory ability than the size factor for the stock return in

Eastern Europe. They substitute the size factor of the Fama-French 3-factor model with the NI/CFO factor.

In 2015, Fama and French add profitability (RMW) and investment (CMA) into their 3-factor model and propose a 5-factor model. RMW represents the difference between the returns on stock portfolios with robust and weak profitability and CMA represents the difference between the returns on the stock portfolios of conservative and aggressive investment firms. Inspired by the q-theory of investment model, Hou et al. (2015) construct the q-factor model which largely summarizes the cross-sectional average stock returns and allows better portfolio valuation. This model explains the expected return of an asset in excess of the risk-free rate with 4 factors: the market excess return (MKT), the difference between the returns on small and large capitalization stocks (ME), the difference between the returns on low and high investment-to-assets stocks (I/A) and the difference between the returns on high and low return-on-equity stocks (ROE).

Apart from the market and size factor, Stambaugh and Yuan (2017) introduce the MGMT factor and PERF factor. These behavior finance factors are constructed by averaging anomaly rankings in contrast to previous research that uses ranking on a single variable. The MGMT factor reflects the management quality of a company and is built on the cluster of anomalies which includes net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment-to-assets. The PERF factor relates more to the performance and is constructed by the cluster of anomalies, including distress, O-score, momentum, gross profitability, and return on assets. Different from the neoclassical factors, the factors in this behavioral model are non-risk factors that represent manifestations of mispricing (Huber and Preissler, 2020).

In the year of 2018, Fama and French further extended their 5-factor model by adding the momentum (UMD) factor. Hou et al. (2020) also augment the q-factor model with the expected growth factor (Eg) to form the q^5 factor model. Evidence from their research suggests that the HML and UMD factors of the Fama-French 6-factor model might be the noisy version of I/A factor and ROE factor. Daniel et al. (2020) build a financing factor (FIN) and introduce a new model that also contains the market risk premium and a post-earnings announcement drift factor (PEAD). Table 1 displays a summary of the most influential modern asset pricing models and the corresponding risk factors.

Table 1. Summary of Influential Modern Asset Pricing Factor Models
(own table created for this thesis)

Model	Abbr.	Research	Risk Factors
Capital Asset Pricing Model	CAPM	Sharpe (1964)	Market (MKT)
Fama-French 3-Factor Model	FF3	Fama and French (1993)	Market (MKT), Size (SMB), Value (HML)
Carhart 4-Factor Model	C4F	Carhart (1997)	Market (MKT), Size (SMB), Value (HML), Momentum (MOM)
q-factor model	q-factor model	Hou, Xue, and Zhang (2015)	Market (MKT), Size (Me), Investment (I/A), Profitability (ROE)
Fama-French 5-Factor Model	FF5	Fama and French (2015)	Market (MKT), Size (SMB), Value (HML), Investment (CMA), Profitability (RMW)
Stambaugh and Yuan 4-factor model	SY4	Stambaugh and Yuan (2017)	Market (MKT), Size (SMB), Management (MGMT), Performance (PERF)
Fama-French 6-Factor Model	FF6	Fama and French (2018)	Market (MKT), Size (SMB), Value (HML), Investment (CMA), Profitability (RMW), Momentum (UMD)
Daniel Hirshleifer and Sun 3-factor model	DHS	Daniel, Hirshleifer, and Sun (2020)	Market (MKT), Financing (FIN), Post-earnings-announcement-drift (PEAD)
q^5factor model	q^5factor model	Hou, Mo, Xue, and Zhang (2020)	Market (MKT), Size (ME), Investment (I/A), Profitability (ROE), Expected Growth (Eg)

2.3 Comparison of Different Asset Pricing Models

There is no doubt that the CAPM is the most essential asset pricing model that has been created to explain the cross-sectional expected returns (Huber and Preissler, 2020). As the theoretical foundation of all the modern asset price models, the empirical performance of CAPM, however, is not satisfying (Fama and French, 1993). The Fama-French 3-factor model largely improves the CAPM’s explanatory power by adding the size (SMB) and value (HML) factors while still maintains simplicity. Since the introduction of this path-breaking model, a research floodgate has been opened and various asset pricing models which contain additional factors have been explored (e.g., Hou et al., 2015; Fama and French, 2015; Stambaugh and Yuan, 2017; Fama and French, 2018; Daniel et al., 2020; Hou et al., 2020). All these modern multi-factor models seem to have good performance with the given testing dataset.

There are three primary approaches in the previous work to compare different asset pricing models. The left-hand-side (LHS) approach (Black, Jensen, and Scholes, 1972; Gibbon, Ross, and Shanken, 1989) is to regress a set of asset returns on the factor variables of the pricing models and to compare the abnormal returns (i.e., the resulting intercepts). The smaller the absolute value of the intercept, the better the factor model explains the return. A major drawback is the dependence on testing portfolio; different testing portfolios will lead to different inferences (Allen and McAleer, 2018; Huber and Preissler, 2020). The right-hand-side approach (RHS) regresses external factors on the factors that are part of the testing subject model or regressing each individual internal factor to other factors that are part of the testing subject model (Fama and French, 2018). Model performance is evaluated by t-statistic for single factor and GRS test for multiple factors. This method cannot be used for the comparison of non-nested models which have completely different factors. For these pricing models, ranking is based on the maximum squared Sharpe ratio. Applying these methodologies, many studies assess and compare the performance of different competing models (e.g., Ammann, Odonia, and Oesch, 2012; Barillas, Robotti, and Shanken, 2019; Huber and Preissler, 2020; Hou et al., 2017, 2019; Fama and French, 2018; Zaremba, Czapkiewicz, Szczygielski, and Kaganov, 2018).

The study of Zaremba et al. (2018) compares the CAPM, the Fama-French 3-factor model, the Fama-French 5-factor as well as the Carhart 4-factor Model with a Polish equity sample, and the Carhart 4-factor model is proven to have the best return explanatory ability in Poland. Ammann et al. (2012) evaluate the 3-factor model of Chen et al. (2011), the Fama-French 3-factor model, and CAPM. They find that the investment-based 3-factor model performs better than the CAPM or Fama-French 3-factor model in explaining return anomalies like asset growth, total accruals, and value effects. Fama and French (2018) adopt the maximum squared Sharpe ratio as a ranking metric to compare both nested and non-nested models. They find that their 6-factor model outperforms the rest and operating profitability factors have better performance than non-cash-based equivalents.

In 2015, Hou et al. compared the q-factor model, Fama-French models, and Carhart models by testing a wide array of variables that have good coverage of all the major anomalies. The testing result reveals that the q-factor model ranks at the top, with the highest Sharpe ratio. This model surpasses the Fama-French and Carhart models in reflecting momentum but not in capturing the operating accrual anomaly and the R&D-to-market anomaly. They comment the HML and UMD factors from the Fama-French models are noisy versions of I/A and ROE factors because the pricing errors of HML and UMD regressed on the q-factors are small and insignificant at 0.06% and 0.13% while large and significant pricing errors are observed by regressing the q-factors to the Fama-French 6-factor model. Hou et al. (2019) further evaluate the empirical performance of the q-factor model, the q^5 factor model and all the other major factor models including the 5-factor and 6-factor model of Fama-French, the 4-factor model of Stambaugh and Yuan (2017), and the 3-factor model of Daniel et al. (2020) with the U.S. stock samples. They find that these different factor models are, in fact, closely related. However, the investment-based q-factor models largely outperform the other models in explaining the maximum number of anomalies and subsume the Fama-French models in spanning regression tests. This result is aligned with their former study and the research of Walkshäusl and Lobe (2014) where a large international stock sample consisting of 40 individual non-US markets is examined.

By incorporating the Sharpe ratio, Barillas et al. (2019) develop the asymptotic pairwise and multiple model comparison methods and measure the efficiency of models including the three Fama-French factor models, the model of Pástor & Stambaugh (2003), the q-factor model and the 4-factor model of Stambaugh and Yuan (2017). They find that the q-factor model dominates all the other models except the Fama-French 6-factor model, which is the best overall performer. However, the q-factor model with the cash profitability factor instead of the ROE factor is superior to the Fama-French 6-factor model. Supported by the pairwise comparison tests (Barillas et al., 2019), Huber and Preissler (2020) perform a regional comparison on factor models from behavioral finance (Daniel et al., 2020; Stambaugh and Yuan, 2017) and neoclassical finance (Sharpe, 1964; Fama and French, 1993, 2015, 2018; Hou et al., 2015; Hou et al., 2020). The result shows in North America, the q-factor models (Hou et al., 2015; Hou et al., 2020) are the winners among all the factor models while in Europe, the 3-factor model of Daniel et al. (2020) and the Fama-French 6-factor model take the lead.

The comparison test results generated from different research vary slightly due to the application of different methodologies, different samples, different testing periods, or different benchmarks. Nevertheless, the majority of evidence from previous work points to the same inference that the Fama-French 6-factor model and the q-factor models have remarkable explanatory power over the average cross-sectional return. The former is motivated by the dividend discount model, and the latter is created with inspiration from Tobin's q investment theory (1969). The empirical results of Hou et al. (2015, 2019, 2020) suggest that the q-factor models can explain all the excess returns of Fama-French models; conversely, the Fama-French models cannot explain the I/A premium of q-factor models despite the presence of the CMA factor. Moreover, the presence of UMD and HML attenuates the ability of the 6-factor model in capturing the Barillas-Shanken value-versus-growth anomalies. The model owners Fama and French (2018) admit that the momentum factor UMD is empirically robust but lacks theoretical motivation and that this factor is included in the model reluctantly, to satisfy insistent popular demand. For the HML, as they highlight in their study (2015), it may be specific to the testing sample; the HML factor becomes redundant in describing the average return when profitability and investment factors are added to their model. As such, the performance of the q-factor models is comparably more robust.

2.4 The Divergent Results of Factor Returns

Existing literature reports different factor return estimates, depending on the empirical model and equity sample. Fama and French (2015) test a broad U.S. equity sample from July 1963 to December 2013. With 2-by-3 sorts, the monthly premiums on SMB, HML, CMA and RMW are 0.29%, 0.37% 0.25% and 0.33% respectively. Except for the size factor of around 0.3%, the returns on other risk factors drop by around 0.1% when using a 2-by-2 sort. The CMA factor is more than halved under a 2-by-2-by-2-by-2 sort. Different ways of factor construction will lead to different empirical results. Hou et al. (2015) report an insignificant size factor return of 0.31% per month on average from January 1972 to December 2012 for the U.S. market. The average return on the investment factor I/A is 0.45% per month, and profitability factor ROE earns a monthly return of 0.58%, much higher than the estimation of Fama and French (2015).

Different equity markets are exposed to different risk factor returns. Huber and Preissler (2020) compute a set of factors over the period from January 1990 to June 2018 across four regions: North America, Europe, the Asia Pacific except Japan, and Japan. The size effect is insignificant in all regions. The average HML premium varies from 0.24% to 0.59%. RMW, CMA, and MOM factors are insignificant at less than 0.1% in Japan whereas Asia Pacific has the highest MOM of 0.85% with a t-value of 3.43. For North America, the average returns on HML, CMA, and RMW are 0.24%, 0.38%, 0.21%, respectively, not far from the estimation of Fama and French (2015). Fama and French also estimate the explanatory returns for these four regions with the 3-factor model in 2012. No size effect is detected from any of these regions as SMB returns are all insignificant and close to zero. Average returns on HML vary from 0.33% in North America to 0.62% in the Asia Pacific. The market premium in Japan is insignificant at -0.12% per month, but significant with a positive value of over 0.56% for other regions. Walkshäusl and Lobe (2014) calculated the monthly factor premium at an international level. Both size and investment effects disappear at this point. Average monthly return on the market factor and value factor are significant at 0.66% (t-statistic=2.35) and 0.72% (t-statistic=3.96).

For the European equity market, asset pricing research is rather fragmented. Walkshäusl (2019) examines the momentum factor on a stock sample over the period from 1990 to 2017, which comprises 15 developed European equity markets. In his study, the momentum premium increases from 1.01 % to 1.09% per month when the firm's fundamental strength levels up. This finding shows a similar pattern as the U.S. equity market; the more the past price performance is congruent with fundamentals, the stronger the momentum effect will be among those companies. Ammann et al. (2012) apply the alternative 3-factor model (Chen et al., 2011) to an integrated sample from 1990 to 2006 containing 10 countries of the European Monetary Union. The result shows that the factors constructed are similar to those corresponding factors in the US market. Evidence from Fletcher's research (2019) suggests that market factor is also the dominant factor in reducing mispricing errors of the individual stocks in the U.K. market, followed with the value factor (HML). Fama and French (2012) estimate an average MKT return of 0.56%, an average HML return of 0.55%, and an average SMB return of -0.06% for the European stock market over the period from November 1990 to March 2011. Foye et al. (2013) employ three factors of Fama and French (1993) to explain the market return in the Eastern European countries that joined the European Union in 2004. They find that the slope coefficients of size (ME) portfolios are also negative and insignificant. This indicates the size factor lacks explanatory power of the risk characteristics which echoes the finding of Fama and French (2012). The SMB factor premium reported by Huber and Preissler (2020) is 0.04% for the

European portfolios along with a t-value of only 0.37, but both the profitability factor return (0.67%) and the growth factor return (0.39%) in Europe are statistically significant.

Growing evidence reveals that some return effects discovered in the US stock market are much weaker or even disappear when conducting an out-of-sample test, i.e., investment factor, profitability factor, and size factor become powerless to explain the return when testing on ex-US samples according to the study of Walkshäusl and Lobe (2014) and Walkshäusl (2019). They regress investment factor and profitability factor on the MKT, SMB, and HML factors and find that no additional information about expected returns is contained in the investment and profitability risk predictor. Analyzing the Polish stock market with the Fama-French 5-factor model and the 4-factor model of Carhart, Zaremba et al. (2018) also identify an insignificant size factor (SMB) and investment factor (CMA) with monthly return premium of 0.14% and 0.06%, t-value of 0.37 and 0.25 respectively.

Overall, previous studies share some valuable patterns despite the divergent empirical results. First, as a universal systematic risk, the size factor is not robust over time or across regions. Second, a factor model has different explanatory power when applied in the different equity market. For example, the Fama-French 3-factor model explains the return better in Europe than in the U.S. (Bauer, Cosemans, and Schotman, 2010). Lastly, some commonly used factors like value factors and profitability factors have significant positive effects on the return of different equity samples, with an average premium ranging from 0.2% to 0.9% per month.

2.5 The Application of Asset pricing model in PE Performance Analysis

2.5.1 Conventional Methodologies of PE Evaluation

Unlike publicly-traded securities or other traditional investment asset classes such as fixed income, private equity is categorized under alternative investments due to its unique characteristics. Neither the interests in a private equity fund nor its portfolio companies are listed on a public exchange; therefore, transparent market prices are not available for the valuation. This illiquid nature of this asset class leads to a particularly challenging situation where performance must be evaluated from observable cash flows alone (Buchner, 2016). As it is relatively easy to calculate and straightforward to interpret, the cash multiple indicators - including total value to paid-in capital

(TVPI) and distribution to paid-in capital (DPI), and internal rate of return (IRR) - become the most widely used performance measurement for PE funds in practice.

TVPI is defined as the sum of distributions and remaining net asset value (NAV) divided by invested amount and DPI equals to distributions received from the fund divided by invested amount. These two cash measurements indicate the multiple of return earned by limited partner (LP) for every unit of the paid-in-capital. Multiples allow quick and easy benchmarking among different individual funds. However, they fail to take the risk and time value of money into account.

The IRR indicates the rate of return based on the cash flows of contributions and distributions over a given period at the breakeven point, wherein the NPV of negative cash flow equals the NPV of positive cash flow. Taking the time value of money and NAV into consideration, the IRR offers a way to compare different investments by analyzing the irregular cash flows. However, this performance indicator is not effective when it comes to assessing mutually exclusive projects which require significantly different amounts of capital or investment durations. And it is often the case that the critical assumption of cash flows being reinvested at the same rate of return is violated in real practice. Phalippou (2008) points out that average IRRs significantly bias upward volatility and performance estimates which are also empirically tested and verified by Phalippou and Gottschalg (2009). In addition, since the IRR denotes the discount rate which is calculated with the NPV of all cash flows equals zero, the result could be multiple-solution or no-solution in some cases.

Sorensen and Jagannathan (2015) suggest that these money-weighted indicators can be easily manipulated if fund managers deliberately choose the timing and magnitudes of cash flows. As absolute measures of performance, IRR and multiples ignore the opportunity cost of private equity investments (Robinson and Sensoy, 2016) and are not directly comparable to the time-weighted returns of public securities. As such, the result of these performance metrics can sometimes generate misleading figures and substantial biases.

To overcome the aforementioned limitations of the absolute performance metrics (Phalippou, 2008; Phalippou and Gottschalg, 2009; Robinson and Sensoy, 2016), relative performance measures such as the public market equivalent (PME) are developed to assess private equity. Long and Nickels (1996) first introduce the methodology of PME which creates a hypothetical investment vehicle that buys and sells public index (S&P500) in a way that mimics irregular cash flows of PE funds. The Long-Nickels PME shows how an equivalent investment with the same investment timings in the public market would have performed. This allows the comparison between the actual IRR of PE funds and the public market index. Furthermore, it greatly reduces the possibility of data manipulation. In the case where an investment greatly outperforms the benchmark index, however, this method will not be applicable due to the negative value yield in the index theoretical investment. In lieu of modifying the NAV of the investment, the PME+ (Rouvinez, 2003) discounts every single distribution with a factor computed so that the NAV of the index investment matches the NAV of the fund. By selling a fixed proportion of the respective PE cash flow instead of an equal amount, PME+ ensures a positive ending balance and fixes the short exposure issue of the Long-Nickels PME whilst preserving the overall cash flow pattern.

While the Long Nickels PME and PME+ compare to actual IRRs of PE funds, the most popular approach, KS-PME proposed by Kaplan and Schoar (2005) returns a direct multiple indicator of the fund performance compared to the public index. This approach does not attempt to adjust the differences caused by systematic risk, but simply implements the PME calculation by discounting contribution and distribution cash flows of a fund at the total return to the public market index (S&P 500) and comparing the resulting value of cash outflows to the invested capital. The KS-PME is defined as the ratio of total discounted outflow cash to total discounted inflow cash:

$$KS - PME = \sum_{t=0}^{T} \frac{D_t}{\prod_{\tau=0}^{t}(1 + r_\tau)} \Bigg/ \sum_{t=0}^{T} \frac{C_t}{\prod_{\tau=0}^{t}(1 + r_\tau)} \tag{5}$$

D_t is the distribution at investment time period t, C_t is the capital call at investment time period t and r_τ is the realized market return from the inception of the fund to the time of the capital call or distribution. A fund with a KS-PME above 1 indicates that the resulting distribution value exceeds the resulting contribution value and that investors get benefit from this investment (Sorensen and Jagannathan, 2015). Another important interpretation is that this fund outperforms the public benchmarking index and vice versa. When Phalippou and Gottschalg (2009) perform private equity benchmarking with the public index S&P 500 with the KS-PME approach, corrections towards sample selection, NAV treatment and performance weights are made in their study, including adding additional European funds, writing off NAV, and weighting by the present value of invested capital instead of committed capital.

Given the high leverage and high systematic risk of buyout funds and venture capital, some literature argues that the performance is overstated in the previous PME research, which simply assumes a beta of 1 (Phalippou and Gottschalg, 2009; Driessen et al., 2012; Robinson and Sensoy, 2016). Phalippou and Gottschalg (2009) adopt the industry and size-matched cost-of-capital to adjust the index returns in order to better reflect the specific risks of PE investments. Phalippou (2014) adds a leverage parameter to the public benchmark index to adjust the market risk for the KS-PME while some other researchers like Robinson and Sensoy (2016) modify the KS-PME by replacing the S&P 500 index with a high-beta public index to reflect the systematic risk. The Generalized Public Market Equivalent (GPME) of Korteweg and Nagel (2016), which redefines the PME as the difference between discounted inflows and outflows for each fund, takes a doubly secured measure to add a risk factor parameter and choose a more suitable index. François, Stoyanova, Shaw, Scott, and Lai (2016) analyze the risk characteristics of portfolio companies in detail and bottom-up construct a size- and sector-adjusted benchmark index to assess the risk-adjusted performance of buyout funds.

Nevertheless, to determine the right benchmark index, that is, to find out the most relevant index to the private equity strategy, is never an easy task for PME methodologies including PME+, KS-PME, and other derived index-based performance measurements. Phalippou (2014) examines the sensitivity of private equity performance to the benchmark alternative. This study shows that the buyout funds underperform by 3.1% per annum when benchmarked to a leveraged small-value index and is similar to that of small-cap stock indices or a passive small-cap mutual fund while public available data present that the buyout funds outperform the S&P 500 by around 5.7% per annum on average. Hence, an inappropriate benchmark index could lead to misleading empirical results. And it is fully possible that fund managers claim a fund to be “top quartile” by selecting benchmarks that position themselves favorably.

2.5.2 Risk and Return Estimation with Asset Pricing Model

Beginning with Cochrane (2005), a growing amount of literature attempts to address the issues of the conventional performance evaluation methodologies by studying adjusted abnormal return and systematic risk characteristics of private capital investment based on the asset pricing frameworks (e.g., Farrelly and Stevenson 2019; Buchner, 2016; Anson 2013; Gredil, Sorensen, and Waller, 2019; Fan, Fleming, and Warren, 2013; Korteweg and Sorensen, 2010; Ang et al., 2018; Driessen et al., 2012 Franzoni, Nowak, and Phalippou, 2012).

In the KS-PME calculation, the discount factor can be viewed as a CAPM with a fixed alpha of 0 and a beta of 1. In this case, the systematic risk cannot be correctly adjusted because the equity premium and risk-free rate are implicitly restricted (Korteweg and Nagel, 2016). In contrast, the asset pricing model-based methodologies can flexibly employ returns on the market factors of a set of public indices to simulate the private equity investment returns.

Buchner (2016) derives the KS-PME with the standard CAPM, the Fama-French 3-factor model, and Pástor-Stambaugh four-factor model. His study illustrates how the contribution and distribution can be decomposed into a risk-free part and other components of corresponding risk factors. While Buchner measures the PME results of PE funds, many other researchers estimate the alpha and beta by using asset pricing factor models as the SDF in the KS-PME metric (Equation 5) and equating PME to 1. The logic behind this methodology is that when the discount rate is the time series of the average realized returns across the set of underlying illiquid investments, the PV of capital distributions is equal to the PV of capital calls (Ang et al., 2018).

Cochrane (2005) tests a cross-sectional CAPM model on the venture capital projects with a maximum likelihood estimate which corrects for selection bias. He analyzes measured returns from investment to IPO/acquisition and identifies the beta of 1.9 for venture capital investments, leaving an alpha of 30%. Anson (2013) reports a contemporaneous beta of 0.4 and a quarterly alpha of 2.6% for PE investment under a single period CAPM model. Market beta almost doubles after adding four lagged beta estimates. Noticing the consistency of the investment patterns and representative investors, Gredil et al. (2019) evaluate cash flows of PE funds from the perspective of financial variables and macroeconomic with two leading consumption-based asset pricing models - the Habit Formation and Long-run Risk. Fan et al. (2013) examine the performance of both buyout and venture capital funds by incorporating a 3-factor model that accounts for size, value, and market. Korteweg and Sorensen (2010) develop a dynamic assessment measurement for the US venture-backed companies with the standard Fama-French 3-factor model. A recent study of Ang et al. (2018) also applies the Fama-French 5-factor model to estimate the time-varying discount rates for a sample of US private equity funds and report a market beta estimate of approximately 1.5.

Driessen et al. (2012) develop a new approach based on an NPV framework, which extends the standard static IRR equation to a dynamic setting using the CAPM and the Fama-French 3-factor model as time-varying discount rates. The risk exposure and the abnormal return are estimated by employing a cross-section General Method of Moments (GMM) estimator to make the NPV as close as possible to zero. In this study, a high market beta is measured for venture capital funds which is close to the value reported by Korteweg and Sorensen (2010). It indicates that venture capital has similar characteristics to those small growth stocks. Furthermore, the alpha is negative even before fees according to both asset pricing models, meaning that the systematic risk is underestimated by investors and the capital cost paid by investors is too high. For buyout funds, the market beta is relatively low and no significant alpha is found. Motivated by the Driessen NPV approach, Franzoni et al. (2012) employ a 4-factor model of Pástor & Stambaugh and find that the beta of the liquidity factor is highly significant. Farrelly and Stevenson (2019) apply the Driessen NPV framework and quantify the risk exposure and abnormal performance of private real estate funds by incorporating the CAPM, the Fama-French 3-factor model, and its four-factor extension including a liquidity factor.

In general, this PE performance benchmarking methodology is superior to other conventional ways from three perspectives: 1) the risk is adjusted by the flexibility of the alpha and beta; 2) instead of using only one benchmark index each time, a multi-factor model can be constructed by various diversified public indices which better represent the market; 3) it reveals more information of the PE investment return since it decomposes the return separately into the abnormal return and the risk premiums of different sources.

2.5.3 Performance Review of Private Equity Funds

It is widely believed that private equity has a higher return than public equity. Given the underlying portfolio companies of private equity funds are not listed, a higher average return is required to compensate for the illiquidity (Cochrane, 2005). Evaluated with the conventional performance matrix, private equity does outperform public securities in general. Harris, Jenkinson, and Kaplan (2014) study cash flow data on 1400 U.S. PE funds and find 20% to 27% outperformance versus the S&P 500 index over the lifetime of the fund. Higson and Rüdiger (2012) also testify that the average buyout fund significantly outperforms the S&P 500 index even after fees. Phalippou (2014) reports an average cross-funds PME of 1.2 which equals an outperformance of 5.7% per annum if spread over an effective holding period of 3.3 years.

When PE fund performance is evaluated in a risk-adjusted way with asset pricing models, the empirical results are more diversified. Estimates from Fan et al. (2013) reveal an underperformance of the U.S. buyout funds with a market beta of around 0.85 to 0.90. Besides, the buyout segment is negatively exposed to size factor and value factor. For venture capital funds, the equity market beta lies even lower at 0.75 with insignificant size and growth exposure. Similarly, Phalippou and Gottschalg (2009) also find that private equity funds underperform the S&P 500 net-of-fees by approximately 3% per year. They calculated a risk-adjusted PI of 0.75 for buyout funds and 0.77 for venture capital funds which means PE funds lost 12% of the value invested in PV terms compared to the public equity investment. With a CAPM, the risk loading of the market factor computed by Franzoni et al. (2012) is slightly less than 1 at 0.95. Driessen et al. (2012) find surprising underperformance for venture capital after fees when benchmarking to small growth stocks. It appears that the underperformance of PE funds may go against the prevailing notion that private equity is risker than public securities due to its high leverage and illiquid nature. A possible explanation is that risk is effectively mitigated through a diversified portfolio or the restricted supply might be overblown (Cochrane, 2005). Another economic interpretation is that the price (i.e., management fees) paid by PE funds to acquire assets is too high (Driessen et al., 2012). In addition, considering many studies where underperformance is observed to have applied the TVE (Thomson Venture Economics) to obtain data, Harris et al. (2014) imply that this underperformance may be partially caused by this data source, where, on occasion, the cash flows are incomplete, and NAVs are incorrect (Higson and Rüdiger, 2012).

The market beta of PE funds evaluated in most of the research is considerably higher. Driessen et al. (2012) report an after-fee market beta of 1.3 for buyout funds. The research of Franzoni et al. (2012) uses the Fama-French 3-factor model as specification and measures a market beta of 1.40. Adding the liquidity factor of Pástor and Stambaugh (2003), the risk loading of market factor decreases to 1.30. The size effect is negative under both specifications but statistically insignificant. The beta of the HML factor ranges from 0.72 to 1.20 and the liquidity factor coefficient is 0.64, making the total risk premium close to 18% per year. In the recent study of Ang et al. (2018), market beta ranges from 1.18 to 1.77 for the buyout segment. The risk exposure of venture investments is stronger with higher market betas between 1.46 to 2.09. Size factor loadings stand at around 0.8 and there is no value effect. The beta coefficient of the liquidity effect is between 0.44 to 0.51 while Buchner (2016) argues the exposure of venture capital return to this factor should be negligible.

The market beta of PE investments is estimated higher than 2 in some literature. The empirical result of Buchner (2014) shows a market beta of 2.2 for buyout funds, benchmarking to the S&P 500. This estimation is also aligned with the work of Axelson (2013) where a market beta of 2.2-2.4 is reported. For venture capital investments, the beta coefficient equals 2.6, which is close to the estimation (2.5 and 2.8) of Driessen et al. (2012) and Korteweg and Sorensen (2010).

Empirical results regarding the alpha magnitude are less conclusive. For buyout investments, Driessen et al. (2012) find a slightly negative alpha, which is statistically insignificant, while Franzoni et al. (2012) find an insignificant positive alpha coefficient of 3.1% under the Fama-French 3-factor model specification. Abnormal return is close to 0 across all models in the research of Ang et al. (2018). Significant alphas are also identified in some studies; for example, 5.6% per annum reported by Fan et al. (2013), 9.3% under the specification of CAPM (Franzoni et al., 2012), and before-fee alphas of 7.0%-8.6% per year estimated by Buchner (2014) and Axelson et al. (2013). For venture capital investments, Fan et al. (2013) report insignificant negative abnormal performance while Driessen et al. (2012) report a strong negative alpha of -12%. And a set of alphas from -0.076% to 2.45% are computed by Korteweg and Nagel (2016) for venture capital funds incepted in different periods. In the research of Ang et al. (2018), abnormal returns across different models are mostly negative but significant. Buchner (2014), on the contrary, detects a strong positive alpha at 8.9% on average. Some studies report even larger before-fee alphas at over 30% per annum (Cochrane 2005, Korteweg and Sorensen, 2010).

The home (U.S.) bias (Karolyi, 2016) is also substantial for PE related research. Most literature in the field of private equity performance evaluation focuses on the U.S., while Europe as the second-largest private equity market receives too little attention. There have been very few studies estimating the risk and return of European PE funds. The estimation result of Buchner (2014) presents an abnormal return of more than 9% for European venture capital. Alpha is lower for buyout funds with a range of 4.1% to 8.9%. The market beta of European venture capital funds (around 1.4) is much lower than that of the U.S. venture investments (around 2.4), whilst European buyout funds (around 2.8) have higher exposure to the market factor than the U.S. buyout funds (around 2.5). By reviewing the limited existing research, one basic pattern is noticeable regarding the European PE fund performance: European private equity funds do not outperform their U.S. peers. Phalippou and Gottschalg (2009) investigate Europe-focused fund performance and find a significant underperformance in comparison with European public equity or US-focused funds. They provide a potential explanation for the low performance: the objective of the investors is not return maximization but could be the stimulation of the local economy, considering a large number of investors are pension funds, insurance companies, or other government-related agencies. Similar evidence is found in the comparative study of Hege, Palomino, and Schwienbacher (2009) which concludes that contraction-related determinants play an essential role in the performance gap. Phalippou (2014) includes European buyout funds into the U.S. sample, and performance indicators remain virtually unchanged as before. This indicates that the European funds might perform equally as the U.S funds.

3. Methodology

3.1 The Q-Factor Model

This thesis adopts the q-factor model of Hou et al. (2015) to compute the return premium of European stocks. Evidence from empirical studies has proven that this model can largely summarize the cross-sectional average equity return and performs well in capturing different categories of anomalies. The q-factor model explains the excess return with four factors: the market factor (MKT), the size factor that uses market capitalization (ME) as the proxy, the investment factor of total assets growth (investment-to-assets, I/A), and the profitability factor which is represented by the return of equity (ROE). The q-factor model is formulated as:

$$E[R_i - R_f] = \beta_{MKT}^{i} E[R_{MKT}] + \beta_{ME}^{i} E[R_{ME}] + \beta_{I/A}^{i} E[R_{I/A}] + \beta_{ROE}^{i} E[R_{ROE}] \qquad (6)$$

where $E[R_i - R_f]$ denotes the expected return of an asset i in excess of the risk-free rate. $E[R_{MKT}]$, $E[R_{ME}]$, $E[R_{I/A}]$, and $E[R_{ROE}]$ denotes expected factor returns, and β_{MKT}^{i} , β_{ME}^{i} , $\beta_{I/A}^{i}$, β_{ROE}^{i} are the respective factor loadings of R_{MKT} , R_{ME}, $R_{I/A}$, R_{ROE}. R_{MKT} is the average market excess return, which equals the value-weighted market return subtracting risk-free rate, R_{ME} is the average spread between the return on a portfolio of small size stocks and the return on a portfolio of big size stocks, $R_{I/A}$ is the average spread between the return on a portfolio of low-investment stocks and the return on a portfolio of high-investment stocks, R_{ROE} is the average spread between the return on a portfolio of high profitability stocks and the return on a portfolio of low profitability stocks.

The market factor is adopted from the CAPM model and is denoted as the return expected from the investor in excess to risk-free return. The size predictor is originally identified by Banz (1981). He notices that for a given market beta level, the average return for the small caps is too high and the average return for the large stocks is too low. He finds that market equity (ME) can strengthen the return explanatory power of the CAPM. Fama and French (1993) further validate this risk predictor and confirm that small stocks earn higher average returns than big stocks. They create the SMB factor and add this variable to their 3-factor pricing model. The investment factor and the profitability factor are largely inspired by the q-theory of Tobin (1969), which hypothesizes that the market value of the company should be equal to the replacement cost. This theory explains the relationship between market value and intrinsic value of a company with the average Q, which can be expressed by the market value of firm capital divided by the replacement cost. Furthermore, this Q ratio serves as a guideline for investment decisions. Based on the implication from Tobin's q-theory, Hou et al. (2015) set up an economic model that demonstrates a negative relation between expected return and investment as well as a positive relation between expected return and a firm's profitability. Specifically, low-investment companies should earn higher expected returns than high-investment companies and low-profitability companies should earn lower expected returns than high-profitability companies. This is because a low discount rate indicates high NPV and thus high investment, and vice versa. However, it should be noted that the investment of a firm is conditional on expected profitability since companies with high ROE tend to invest more than less profitable companies. The betas are risk exposures of associated risk factors which can be understood as the sensitivity of returns reacting to risk variables.

Instead of using individual stocks for the return computation, the q-factor construction employs a triple 2-by-3-by-3 sort on market capitalization, investment-to-assets, and ROE. This standard sorting methodology was first developed by Fama and French (1993, 1996). By grouping stocks into portfolios, the idiosyncratic risk can be reduced to a large degree because diversified stocks can offset the idiosyncratic volatility of each individual stock in a portfolio (Fama and MacBeth, 1973), and, consequently, the risk premia can be estimated more precisely (Ang, Liu, and Schwarz, 2020).

3.2 The GMM Estimator of Driessen et al. (2012)

In this study, the methodology applied to perform the PE fund benchmarking is a modified version of the NPV-based model. It is introduced by Driessen et al. (2012) and can be viewed as the standard IRR calculation in an NPV framework extended with a dynamic return factor model. The goal of applying this model, however, is not calculating the IRR but to estimate the optimal alpha and beta coefficients of the fund investment.

Assuming each investment project contains N PE funds, for undying fund i (i = 1, …, N), there are cash flows generated throughout the lifespan from inception date t_{i0} to liquidation date t_{il}. T_i is the period of the final cash flow. Contribution from the investors at time t of fund i is denoted as C_{it}, and distribution to the investor at time t of fund i is denoted as D_{it}. IRR is the discount rate of fund i that makes the NPV of all the cash flows including outflows and inflows equal to zero. The IRR can be calculated by solving:

$$NPV = \sum_{t=t_{0i}}^{T_i} \left[\frac{D_{it} - C_{it}}{(1 + IRR_i)^t}\right] = 0 \tag{7}$$

Driessen et al. (2012) replace the constant discount rate IRR_i in Equation (7) with a linear one-factor return model (i.e., the CAPM) which can be presented as $R_{f,t} + \alpha_i + \beta_i R_{m,t}$, where $R_{f,t}$ is the risk-free rate at time *t* and $R_{m,t}$ is the realized market return in excess to the risk-free rate at time *t*. This allows a dynamic NPV framework where the discount rate varies over time:

$$\sum_{t=t_{0i}}^{T_i} \left[\frac{D_{it} - C_{it}}{\prod_{s=t_{0i}+1}^{t} (1 + R_{f,s} + \alpha_i + \beta_i R_{m,s})} \right] = 0 \tag{8}$$

The given data, including contributions, distributions, risk-free rate, and market excess return are not sufficient to compute α_i and β_i with this single equation. Therefore, it is necessary to assume a common parametric structure across all funds. Under this assumption, $\alpha_i = \alpha$, $\beta_i = \beta$, α and β coefficients can now be estimated with the below least-square optimization by making the NPV of all the underlying funds as closest to 0 as possible.

$$\min_{\alpha,\beta} \sum_{i=1}^{N} [NPV_i(\alpha, \beta)]^2 \tag{9}$$

where

$$NPV_i(\alpha, \beta) = \sum_{t=t_{0i}}^{T_i} \left[\frac{D_{it} - C_{it}}{\prod_{s=t_{0i}+1}^{t} (1 + R_{f,s} + \alpha_i + \beta_i R_{m,s})} \right] \tag{10}$$

This method can be perceived as GMM (Generalized Method of Moments) estimator. An important contribution of this method is that it only requires fund-level cash flow data to identify alpha and beta and does not need the assumption of 1-period return estimation which is almost impossible for the non-traded assets like private equity (Driessen et al., 2012).

This approach simply discounts all the fund cash flows back to the fund inception date, which causes an upward bias for the alpha estimation. To alleviate the exploding alpha issue and reduce variance, this study modifies this framework with the guidance of Tausch (2020) to calculate an average NPV. First, all the cash flows of fund i are discounted to period τ, NPV of fund i at time τ can be presented by the below equation:

$$NPV_{\tau,i} = \sum_{t=t_{0i}}^{T_i} \left[\frac{D_{it} - C_{it}}{\prod_{s=t_{0i}+1}^{t} (1 + R_s)} \right] * \prod_{s=t_{0i}+1}^{\tau} (1 + R_s) \qquad (11)$$

And the average value of all the NPVs is calculated as:

$$\mu(NPV_i) = \frac{1}{T_i} \sum_{t=t_{0i}}^{T_i} NPV_{\tau,i} \qquad (12)$$

By factoring realized market return with a specification of the q-factor model, R_s equals $R_f + \alpha + \beta_{MKT}R_{MKT} + \beta_{ME}R_{ME} + \beta_{I/A}R_{I/A} + \beta_{ROE}R_{ROE}$. It should be noted, however, that with too many factors as input, the modelling will suffer from the overfitting error which leads to bad out-of-sample performance. When a model is too closely fit a limited set of data points which normally contains some certain mistakes and random noise, it is likely that the pattern predicted is just chance occurrences and does not fit to other samples. In hopes that model stability out-of-sample can be improved, imposing parameter parsimony is crucial (Gu, Kelly, and Xiu, 2020). Thus, this thesis applies a specification that comprises a maximum of two variables: the market factor (R_{MKT}) along with a q-factor other than market factor (R_{ME} , $R_{I/A}$ or R_{ROE}). As such, R_s is replaced by $R_f + \beta_{MKT}R_{MKT}$, $R_f + \alpha + \beta_{MKT}R_{MKT}$, $R_f + \beta_{MKT}R_{MKT} + \beta_{ME}R_{ME}$, $R_f + \beta_{MKT}R_{MKT} + \beta_{I/A}R_{I/A}$, or $R_f + \beta_{MKT}R_{MKT} + \beta_{ROE}R_{ROE}$. β_{ME} , $\beta_{I/A}$ and β_{ROE} are denoted by β_q, the GMM estimator is formulated as below:

$$\min_{\alpha,\beta_{MKT},\beta_q} \sum_{i=1}^{N} \left[\mu(NPV_i)(\alpha, \beta_{MKT}, \beta_q)\right]^2 \qquad (13)$$

4. Research and Data Analysis

4.1 Construction of European Index

The goal is to base the q-factor return calculation on a reasonably representative European equity sample which contains as many listed companies and covers as many equity markets as possible. Since most of the European countries have their own stock exchanges, and some countries like Germany and Spain have more than one; it would be massive manual work to retrieve data from each exchange platform and select stocks. Therefore, this study collects companies from 45 most recent and active benchmark equity indices, including a special Thomson Reuter Index - Europe 2500+, Euro Stoxx indices, S&P 350, FSTE Euro Indices, etc. These indices are sourced from

Thomson Reuters Datastream. However, historical companies deleted from those indices are not available in Datastream. In order to alleviate survivorship bias, dead companies[4] which have been deleted from the Euro Stoxx indices over time are added back to the database. Table 2 presents the comprehensive index list that is used to construct the European index sample.

Table 2. Benchmark Equity Indices of Europe

Name	Symbol	Market	Source
AEX ALL SHARE	LNLALSHR	Netherlands	Euronext Amsterdam
AEX INDEX	LAMSTEOE	Netherlands	Euronext Amsterdam
Athex Composite Index	LGRAGENL	Greece	Athens Stock Exchange
ATX Index	LATXINDX	Austria	Wiener Boerse
BEL 20	LBGBEL20	Belgium	BEL Group
BUDAPEST (BUX)	LBUXINDX	Hungary	Budapest Stock Exchange
CAC 40 Constituents	LFRCAC40	France	Euronext Paris
CROBEX CONSTITUENTS	LCTCROBE	Croatia	Crobex
DAX Index Constituents	LDAXINDX	Germany	Deutsche Boerse
EURO STOXX	LDJEURST	Euro	STOXX
Euronext 100	LEUNX100	Europe	Euronext
Euronext 150	LEUNX150	Europe	Euronext
Europe 2500+	G#LTOTMKER	Europe	Thomson Reuters
FTSE 100 CONSTITUENTS	LFTSE100	United Kingdom	FTSE
FTSE 250 CONSTITUENTS	LFTSE250	United Kingdom	FTSE
FTSE MIB	LFTSEMIB	Italy	FTSE
FTSE/ATHEX 20	LFTASE20	Greece	Athens Stock Exchange
FTSEUROFIRST 100 E	LFTEFC1E	Europe	FTSE
FTSEUROFIRST 80 E	LFTEF80E	Europe	FTSE
IBEX 35 INDEX	LIBEX35I	Spain	BME, Spanish Exchanges
ISEQ All-Share	LISEQUIT	Ireland	Irish Stock Exchange
LFTEU100	LFTEU100	Europe	FTSE
MADRID SE GENERAL	LMADRIDI	Spain	Bolsas y Mercados Espanoles, BME (Spain)
MDAX Index	LMDAXIDX	Germany	Deutsche Boerse
MSCI EUROPE - DAILY	LMSEROPD	Europe	MSCI
OMX COPENHAGEN (OMXC)	LCOSEASH	Denmark	Nasdaq OMX
OMX COPENHAGEN (OMXC20)	LDKKFXIN	Denmark	Nasdaq OMX
OMX HELSINKI (OMXH)	LHEXINDX	Finland	Nasdaq OMX
OMX ICELAND ALL-SHARE	LICEXALL	Iceland	Nasdaq OMX
OMX STOCKHOLM (OMXS)	LSWSEALI	Sweden	Nasdaq OMX
OMX STOCKHOLM 30 (OMXS30)	LSWEDOMX	Sweden	Nasdaq OMX

[4] Dead companies of the Euro Stoxx indices are retrieved from www.stoxx.com.

Oslo Exchange All-share Index_GI	LOSLOASH	Norway	Oslo Bors
PORTUGAL PSI ALL-SHARE	LPOPSIGN	Portugal	Euronext Lisbon
PORTUGAL PSI-20	LPOPSI20	Portugal	Euronext Lisbon
Prague SE PX	LCZPXIDX	Czech Republic	Prague Stock Exchange
Prime All Share	LPRIMALL	Germany	Deutsche Boerse
S&P Europe 350	LSPEU350	Europe	STOXX
SBF 120 Constituents	LFSBF120	France	Euronext Paris
SBI TOP INDEX	LSLOETOP	Slovenia	Ljubljana Stock Exchange
SOFIX_WEIGHTING	LBSSOFIX	Bulgaria	-
STOXX 50	LDJSTO50	Europe	STOXX
STOXX 600	LDJSTOXX	Europe	STOXX
Swiss Market index	LSWISSMI	Switzerland	SWX Swiss Exchange
WARSAW Generel	LPOLWIGI	Poland	Warsaw Stock Exchange

To sort out the integrated European stock market sample, the following filters (Huber and Preissler, 2020) are applied: 1) Security type must be common equity; 2) Only major stock is included in the case of multi-listing firms; 3) Only primary quotations of security are considered. 4) Firms that are not located and listed in the associated countries are excluded. After removing duplication, the target European Index is constructed. It consists of 5777 companies across all industry sectors, including energy, materials, industrials, consumer discretionary, consumer staples, health care, financials, information technology, communication service, utilities, and real estate. The underlying companies are from 34 European equity markets: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Liechtenstein, Lithuania, Luxembourg, Malta, Monaco, Netherlands, Norway, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, and United Kingdom.

4.2 Data Source and Raw Data Specification

Thomson Reuter Datastream and the CRSP (Center for Research in Security Prices) are the most popular data sources in recent empirical research on asset pricing. The CRSP database contains high-quality end-of-day historical data, but it only covers U.S. securities, including stock, index, treasury, and mutual funds database and therefore it is usually maintained for academic research of the U.S. equity market. On the other hand, Thomson Reuter Datastream provides services worldwide. It has a deep and broad coverage in terms of the number of markets and the number of securities in each market (Ince and Porter, 2006). Hence, this European data-based study mainly uses Thomson Reuter Datastream as the primary data source.

Monthly return index (RI) and security-level accounting data including year-end market capitalization, year-end total assets, and annual return on equity (ROE) of 5777 companies are retrieved from Thomson Reuters Datastream. Since market data for most stock markets are very limited in earlier years and accounting data are usually not available before 1980 (Jacobs and Müller, 2020), the sample period starts from January 1980 and extends to December 2019. Market capitalization and total assets are denominated in euros.

According to the data specification in Datastream, the return index is defined as the value growth of a share holding over a specified period with the assumption that dividends are reinvested at the closing price of the ex-date. Before the year of 1988, RI is calculated using annualized dividend yield as below:

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}}(1 + \frac{DY_t}{100} * \frac{1}{N}) \tag{14}$$

where RI_t denotes the return index on day t, RI_{t-1} denotes the return on day $t-1$, PI_t denotes the price index on day t, PI_{t-1} denotes the price index on day $t-1$, DY_t denotes the dividend yield on day t, and N denotes the total working days in one year. Here it is assumed as 260 business days without taking market holidays into account.

From 1988 onwards, thanks to the availability of detailed dividend payment data, a more precise methodology is in place, in which the closing price as of ex-date is adjusted with the discrete quantity of dividend payment.

$$RI_t = RI_{t-1} * \frac{P_t}{P_{t-1}} \tag{15}$$

In the case that t is ex-date, the formula for RI calculation is presented as below:

$$RI_t = RI_{t-1} * \frac{P_t + D_t}{P_{t-1}} \tag{16}$$

where P_t is the price on ex-date, P_{t-1} is the price of the previous day, and D_t is the respective dividend payment. Gross dividend is applied in the calculation of RI, which means tax and commission of the reinvestment process are ignored. And prior to 2004, the RI of UK companies contains a tax credit on the announced dividend, and this part is included in the RI calculation.

Market capitalization is equal to the market price times the common share issued. For European companies, market price applies the closing price of the company stock as of fiscal year-end. Preferred shares are out of the calculation.

Total assets are the sum of total current assets and total non-current assets, including cash and equivalents, receivables, inventory, investments, intangibles, equipment, plants, and other assets. For banks and other financial companies, it also comprises net loans, custody securities, customer liability on acceptances. Contingent liability and deferred taxes are excluded. Starting from October 2012, trust business assets are categorized into total assets while other items such as foreign currency translation gain/loss, bad debt/loan losses, treasury stock, investment in own bonds are excluded.

Return on equity[5] is a profitability measure calculated by net income subtracting bottom line preferred dividend requirement and then divided by the average of 1-year-lagged book equity and current year's book equity.

Most companies only release year-end financial statements, quarterly and monthly accounting data are generally quite limited, thus market capitalization, and total assets are year-end balance for these companies and ROE is on an annual basis.

This research adopts the EONIA rate (euro overnight index average) and the U.S. one-month Treasury bill (T-bill) rate as the proxies of the risk-free rate. Daily annualized EONIA rate from January 1999 to December 2019 is extracted from the Statistical Data Warehouse of the European Central Bank which is published by the European Money Market Institute. The EONIA rate is the closing rate for the overnight maturity calculated as the euro short-term rate plus a fixed spread of 8.5 basis points[6]. And the

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[5] ROE = (Net Income – Bottom Line Preferred Dividend Requirement) / Average of Last Year's and Current Year's Common Equity * 100%

[6] Cited from the data specification in the Statistical Data Warehouse of the European Central Bank: https://www.ecb.europa.eu/

monthly interest rate of the U.S. one-month T-bill is sourced from the Kenneth R. French data library.

4.3 Dataset Preparation and Screening Process

To prepare the construction of four q-factors, monthly market return, market capitalization, investment-to-assets, and return on equity data on individual security level are required. Therein, monthly market return and investment-to-assets are further calculated based on the raw data. Additionally, to reduce data noise and remove extreme outliers, all these data need to be trimmed and filtered properly before the factor construction process.

Monthly market return R_{MKT} of company i is calculated as total return of current month t divided by 1-month lagged total return minus 1:

$$R^i_{MKT} = \frac{RI_t}{RI_{t-1}} - 1 \tag{17}$$

Monthly return is set as missing if RI_t or RI_{t-1} is 0 or N/A. In order to remove unreasonable outliers, returns larger than 990% is removed from the dataset (Huber and Preissler, 2020). In addition, monthly returns are winsorized at the 10% and the 90% level to exclude extreme observations from the data.

Investment-to-assets (I/A) of individual security i is measured as the annual change in total assets (TA) divided by 1-year-lagged total assets (Hou et al., 2015).

$$I/A_i = \frac{TA_t}{TA_{t-1}} - 1 \tag{18}$$

Applying the same rule as the market return, I/A is set as missing when TA_t or TA_{t-1} is 0 or N/A. I/A larger than 100 is deleted from the database.

To ensure that the results are not driven by the smallest or the most illiquid companies (Jacobs and Müller, 2020), market capitalization less than 1 million Euros is filtered out. Instead of using income before extraordinary items divided by 1-quarter-lagged book equity (Hou et al., 2015), ROE is adjusted as the difference between net income and bottom line preferred dividend requirement divided by the average shareholder's equity. Given the fixed dividend, the preferred share has a closer feature to the debt, thus subtracting the preferred dividend from the net income would be more precise and conservative to estimate ROE. And from the accounting perspective, it is considered as the best practice to calculate ROE based on average equity over a period because equity is a running balance at a given point while net income is concerned about a particular time frame. As part of the trimming process, ROE with an absolute value larger than 1000 is removed.

4.4 Portfolio Formation

In empirical finance, the most natural methodological approach to investigate the relationship between the excess return and a certain characteristic is to sort observed returns according to the characteristic value, divide stocks into portfolios by the characteristic, and compare the average return differences across the portfolios (Cattaneo, Crump, Farrell, and Schaumburg, 2019). This study sorts the integrated European stock sample based on the median of market capitalization (ME), and the percentile of investment-to-assets (I/A) and return on equity (ROE). ME, I/A, and ROE represent the return characteristic of size, investment, and profitability, respectively. Following the practice of Hou et al. (2015), a triple 2-by-3-by-3 sort on these three factors is built to form 18 portfolios, and each stock with valid values is assigned to one of these portfolios. It is important to note that some resulting portfolios may only contain a few stocks in earlier years; thus, idiosyncratic risk impact cannot be fully eliminated (Bauer et al., 2010).

Specifically, European stocks are split into two groups by the median size (ME) of the sample at the end of each month, a big-sized group B, and a small-sized group S. After controlling the size variable, at the end of June of each year, all stocks are split into three groups using the breakpoints at the 30th and 70th percentiles of the ranked value of I/A, a low-investment group L, a medium-investment group M and a high-investment group H. And similarly, stocks are then broken up into three groups based on the breakpoints of 30th and 70th percentiles of the ranked values of ROE, a low-profitability group L, a medium-profitability group M and a high-profitability group H. The intersection of 2 ME, 3 I/A, and 3 ROE groups create 18 portfolios as presented in table 3.

Table 3. Portfolios Formation by ME, I/A and ROE

This table presents the portfolio formation by ME, I/A, and ROE. The European stock samples are split into two groups by the median size at the end of each month, a big-sized group B, and a small-sized group S. After controlling the size variable, at the end of June of each year, all stocks are split into three groups using the breakpoints at the 30th and 70th percentiles of the ranked value of I/A, a low-investment group L, a medium-investment group M and a high-investment group H. And similarly, stocks are then broken up into three groups based on the breakpoints of 30th and 70th percentiles of the ranked values of ROE, a low-profitability group L, a medium-profitability group M and a high-profitability group H.

ME	I/A			ROE
	Low	Medium	High	
	SLL	SML	SHL	Low
Small	SLM	SMM	SHM	Medium
	SLH	SMH	SHH	High
	BLL	BML	BHL	Low
Big	BLM	BMM	BHM	Medium
	BLH	BMH	BHH	High

The size groups are rebalanced at the end of each month. However, due to the monthly market capitalization data unavailability of most companies, the size portfolios will not change significantly throughout a year. The investment groups and the profitability groups are rebalanced at the end of June of each year. As such, a total of 18 portfolios are revised on a monthly basis and this allows stocks to move freely from one portfolio to another (Asad, Khalid, and Faraz, 2017).

4.5 Factor Definition and Construction

During the factor construction, the equal weighting of different sized stocks could cause underestimation of small stock premiums and overestimation of big stock premiums (Fama and French, 2018). Considering this limitation of the equal-weighting approach, this study calculates value-weighted monthly returns for each portfolio by assigning weights to the market capitalization of the underlying stocks. The big sized companies contribute more to the portfolio return because of the larger weight.

The size factor (R_{ME}), the investment factor ($R_{I/A}$), and the profitability factor (R_{ROE}) are replicated each month as per the q-factor construction procedure of the U.S. market (Hou et al., 2015). R_{ME} (small-minus-big) is the difference between the simple average return of the 9 small-sized portfolios (SLL, SLM, SLH, SML, SMM, SMH, SHL, SHM, SHH) and the simple average return of 9 big-sized portfolios (BLL, BLM, BLH, BML, BMM, BMH, BHL, BHM, BHH). $R_{I/A}$(low-minus-high) is the simple average return on the 6 low I/A portfolios (SLL, SLM, SLH, BLL, BLM, BLH) less the simple average return on the 6 high I/A portfolios (SHL, SHM, SHH, BHL, BHM, BHH). And R_{ROE} (high-minus-low) is the simple average return of the 6 high ROE portfolios (SLH, SMH, SHH, BLH, BMH, BHH) less the simple average return of 6 low ROE portfolios (SLL, SML, SHL, BLL, BML, BHL).

The market factor MKT is constructed as the value-weighted market return R_{MKT} of the whole sample, each month, subtract risk-free rate. Before the year of 1999, good risk-free rate proxies were extremely hard to obtain for the Europe region. It is more common to consider the German government bond yield as the risk-free return for Europe in the 80s since Germany has been the largest and most stable economy in Europe. But given the high volatility of the monthly rate calculated based on the total return of government index, this thesis follows the practice of Fama and French (2012), the monthly interest rate of the U.S. one-month T-bill is taken as the proxy of the risk-free rate. Starting from January 1999, a new risk-free benchmark - EONIA rate is available for the Eurozone, which better fits the integrated European sample. Therefore, from 1999 onwards, this research computes the MKT factor as value-weighted market returns less monthly EONIA rate[7].

4.6 Empirical Result - Factor Returns in Europe

4.6.1 Summary Statistics of Portfolio Return

The average value-weighted returns of 18 size-investment-profitability portfolios from January 1981 to December 2019 are presented in Table 4. The average monthly returns of European stocks range from -0.43% to 1.53% with moderately high volatility (i.e., SD varies from 1.82 to 3.31). A rough pattern can be observed from this summary statistics: small-sized stock samples have lower returns than big-sized stock samples, high-investment portfolios earn higher yields than low-investment portfolios and high-profitability stock groups perform better than low-profitability stock groups. Group BHH yields the highest return among the 18 portfolios and group SLL has the worst performance. This pattern will be further elaborated in the next sections.

[7] The annualized EONIA rate is converted into monthly return with the below formula:
(1 + Annual Return) ^ (1/12) - 1

Table 4. Summary Statistics of Portfolio Return: Jan. 1981-Dec. 2019

This table presents the summary statistics of 18 size-investment-profitability portfolios over the period from January 1981 to December 2019 (468 months in total), including average monthly return (in %), standard deviation, maximum value, and minimum value. The return is calculated with a value-weighting approach. Portfolio groups are rebalanced each month.

Portfolio	Obs.	Mean	SD	Min	Max
SLL	468	-0.430127	2.037992	-7.025964	6.719113
SLM	468	0.201499	2.111736	-6.963643	7.209793
SLH	468	0.546364	2.409217	-9.593948	7.447736
SML	468	-0.130265	2.161165	-6.297413	6.519631
SMM	468	0.365925	1.826309	-7.562107	5.555336
SMH	468	0.865933	2.214726	-9.651204	7.652281
SHL	468	0.186821	2.589319	-7.432949	10.362716
SHM	468	0.563206	2.217260	-8.435977	7.914234
SHH	468	1.076558	2.130186	-5.721589	7.065789
BLL	468	0.474389	2.954447	-8.509526	8.541627
BLM	468	0.987967	3.051854	-8.309852	7.863341
BLH	468	1.087195	3.085785	-9.949339	8.605303
BML	468	0.740273	3.273207	-8.459382	9.682080
BMM	468	1.084014	2.944209	-8.830490	8.721118
BMH	468	1.241792	3.001433	-9.526617	8.563940
BHL	468	0.761496	3.315250	-8.707692	10.950779
BHM	468	1.182004	3.082783	-8.539770	10.826970
BHH	468	1.530552	3.048370	-8.995912	8.873942

4.6.2 Q-Factor return Premium

Table 5 exhibits the summary statistics of monthly factor returns in Europe and the U.S. over the sample period from January 1981 to December 2019. The time-series of monthly factor premiums in Europe are plotted in Figure 1, and Figure 2 presents the time-series of annual factor premiums.

As shown in Panel A of Table 5, all the average monthly q-factor returns are statistically significant in Europe with t-values larger than 2 during the sample period. The average risk-free rate in Europe is 0.39 % per month. It goes all the way down from 1% at the beginning of the 1980s to below zero in 2010s after the financial crisis due to the structural and cyclical factors[8]. The need to ease financing conditions urges the European central bank to adopt a negative interest rate policy so that the consumption can be stimulated, and the economy can be boosted in Europe. It is also designed to weaken the euro and encourage exports.

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[8] Cited from the public issue of the European Central Bank: https://www.ecb.europa.eu/

Table 5. Summary Statistics of Factor Premiums

Panel A presents the summary statistics of the time-series factor premiums (in %) from January 1981 to December 2019 in Europe, 468 months in total. The monthly risk-free rate after 1999 is calculated from the annualized EONIA rate, and the one-month U.S. T-bill rate is used as proxy pre-1999. The market factor R_{MKT} is constructed as the value-weighted market return of the whole sample each month subtract risk-free rate. R_{ME} (small-minus-big) is the difference between the simple average return of the 9 small size portfolios and the simple average return of 9 big size portfolios. $R_{I/A}$(low-minus-high) is the difference between the simple average return on the 6 low I/A portfolios and the simple average return 6 high I/A portfolios. And R_{ROE} (high-minus-low) is the difference between the simple average return of the 6 high ROE portfolios and the simple average return of 6 low ROE portfolios. Panel B presents the summary statistics of the factor premiums (in %) from January 1981 to December 2019 in the U.S. region. The monthly factor returns data are retrieved from http://global-q.org/factors.html which are released and maintained by Hou et al.

Panel A: Factor Premiums of Europe

Return Factor	Mean	SE	t-value	Median	SD
R_F	0.386298	0.111377	3.468371	0.169464	2.409461
R_MKT	0.692745	0.149401	4.636828	0.943376	3.232031
R_ME	-0.652654	0.065815	9.916428	-0.656647	1.423805
R_I/A	-0.408089	0.058126	7.020801	-0.404669	1.257450
R_ROE	0.794235	0.053047	14.972289	0.823493	1.147577

Panel B: Factor Premiums of the U.S.

Return Factor	Mean	SE	t-value	Median	SD
R_F	0.329318	0.012971	25.389704	0.326800	0.280596
R_MKT	0.624264	0.201008	3.105670	1.030500	4.348464
R_ME	0.153056	0.134965	1.134039	0.020400	2.919739
R_I/A	0.323187	0.088376	3.656961	0.288700	1.911859
R_ROE	0.519539	0.116626	4.454725	0.626450	2.523016

The market factor produces an average monthly return of 0.69% in the equity market of Europe. It is slightly higher compared to the market premium in the U.S. stock market (0.62%). The median of market factor premium stands at 0.94% per month, which means that some underperformed stocks drag down the average return to a lower level. And the standard deviation is substantially bigger than the ones of other factors. As can be seen in Figure 1 and Figure 2, the market factor appears to have high volatility which roughly follows the megatrend of the market. Market effect hits bottom during the recession and financial crisis in 1987, 1990, and 2008 with monthly returns less than -5%.

In Figure 2, one can see the annual size premium is less than zero most of the time. The performance of this factor during the period of 1981 to 2000 is volatile but tends to be more stable after 2005. The European market delivers a negative average size effect of -0.65% per month, significantly different from zero with a t-value of 9.9. This indicates a contradictory phenomenon as North America that European stocks with smaller market capitalization do not necessarily have higher returns than those with larger market capitalization. This result, however, is aligned with some of the recent research (Bauer et al., 2010; Fama and French, 2012; Foye et al., 2013; Van Dijk, 2011). Bauer et al. (2010) report a negative size return of -0.093% and Fama & French (2012) report an average size premium of -0.06% for the European stock market over the period of 1991-2010. Foye et al. (2013) also estimate a negative size factor return in Eastern European Countries. As pointed out by VanDijik (2011), even though the size factor arises endogenously as compensation for systematic risk in numerous theoretical pricing models, it has been an ongoing debate about the size premium. Some empirical studies report positive size factor returns (Fama and French, 2015; Hou et al., 2015, 2020; Huber and Preissler; 2020), some assert that this factor does not have explanatory power for the return in some certain regions (Fama and French, 2012; Walkshäusl and Lobe, 2014; Zaremba et al., 2018), some disprove its persistence and validity and some even argue that it has disappeared since the early 1980s (Van Dijk, 2011). The incremental effect of the size factor is not robust even in North America. Hou et al. (2015) find that the anomaly-capturing capability of the size effect is rather limited. The mean absolute alpha averages across the deciles only changed by 0.0001 from 0.11% to 0.12% when the size factor is dropped from the q-factor model.

The investment factor is less volatile and appears to lag the market factor around one quarter to one year as presented in Figure 1 and Figure 2. A surprising outcome is that the I/A factor also earns a significant negative premium in the European market at -0.41% per month on average. The investment factor estimated by Walkshäusl and Lobe (2014) at the international level is also negative. However, according to Hou et al. (2015), return decreases when the I/A ratio goes higher because high-investment stocks earn lower returns than low-investment stocks. Theoretically, companies make investment decisions based on the discount rate. When the cost of capital is lower, the discount rate is lower and thus leads to higher NPV of the future cash flows (Ammann et al. 2012). From the capital budgeting perspective, companies invest more if the NPV is high; therefore, high investment implies a low discount rate and low expected return, and that, all else equal, there is less investment when the discount rate and the expected return are higher. Nevertheless, in practice, mature companies or the growth companies that are scaling business tend to invest more regardless of the discount rate. The expansion plans are usually considered as a signal that the firms are operating well and thus attracting more investment. From an economic perspective, higher demand

Figure 1. Time-Series Monthly Q-Factor Premiums of European Equity Market

This Figure plots the monthly q-factor returns and the risk-free rate of the European stock market from January 1981 to December 2019 (in %), 468 months in total. The monthly risk-free rate after 1999 is calculated from the annualized EONIA rate, and the one-month U.S. T-bill rate is used as proxy pre-1999. The market factor MKT is constructed as the value-weighted market return R_{MKT} of the whole sample each month subtract risk-free rate. R_{ME} is the difference between the simple average return of the 9 small size portfolios and the simple average return of 9 big size portfolios. $R_{I/A}$ is the difference between the simple average return on the 6 low I/A portfolios and the simple average return 6 high I/A portfolios. And R_{ROE} is the difference between the simple average return of the 6 high ROE portfolios and the simple average return of 6 low ROE portfolios. The grey bands indicate economic financial crisis and NBER recession indicators: 1981.7-1982.11, 1986.10-1986.12, 1987.9-1987.11, 1989.9-1989.12, 1990.7-1991.3, 1991.8-1992.12, 1994.7-1994.10, 1997.5-1997.9, 1998.8-1998.10, 2000.2-2000.4, 2001.3-2001.11, 2005.8-2005.11, 2007.9-2009.6, 2010.8-2010.10, 2012.5-2012.7, 2015.3-2015.12.

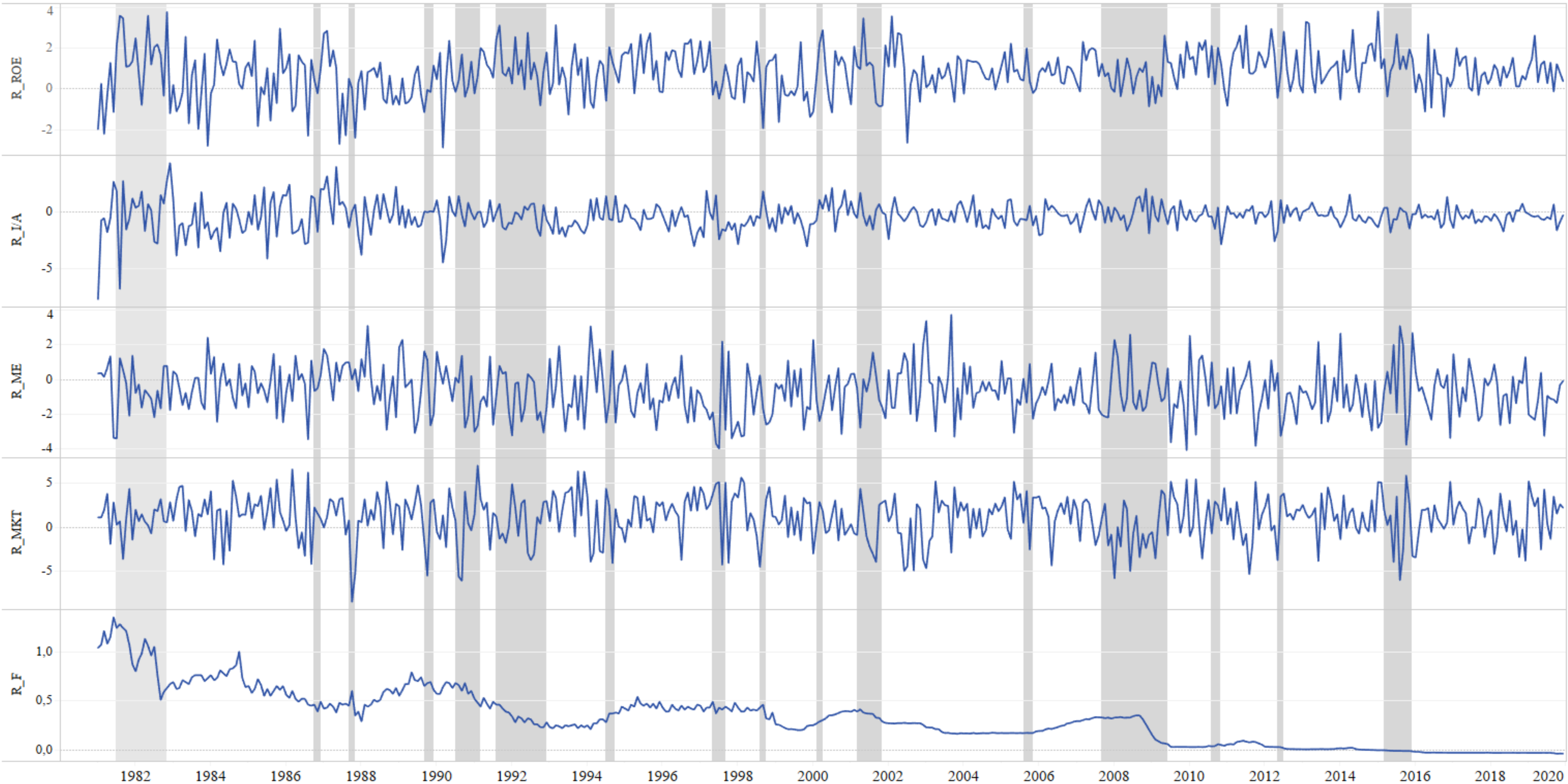
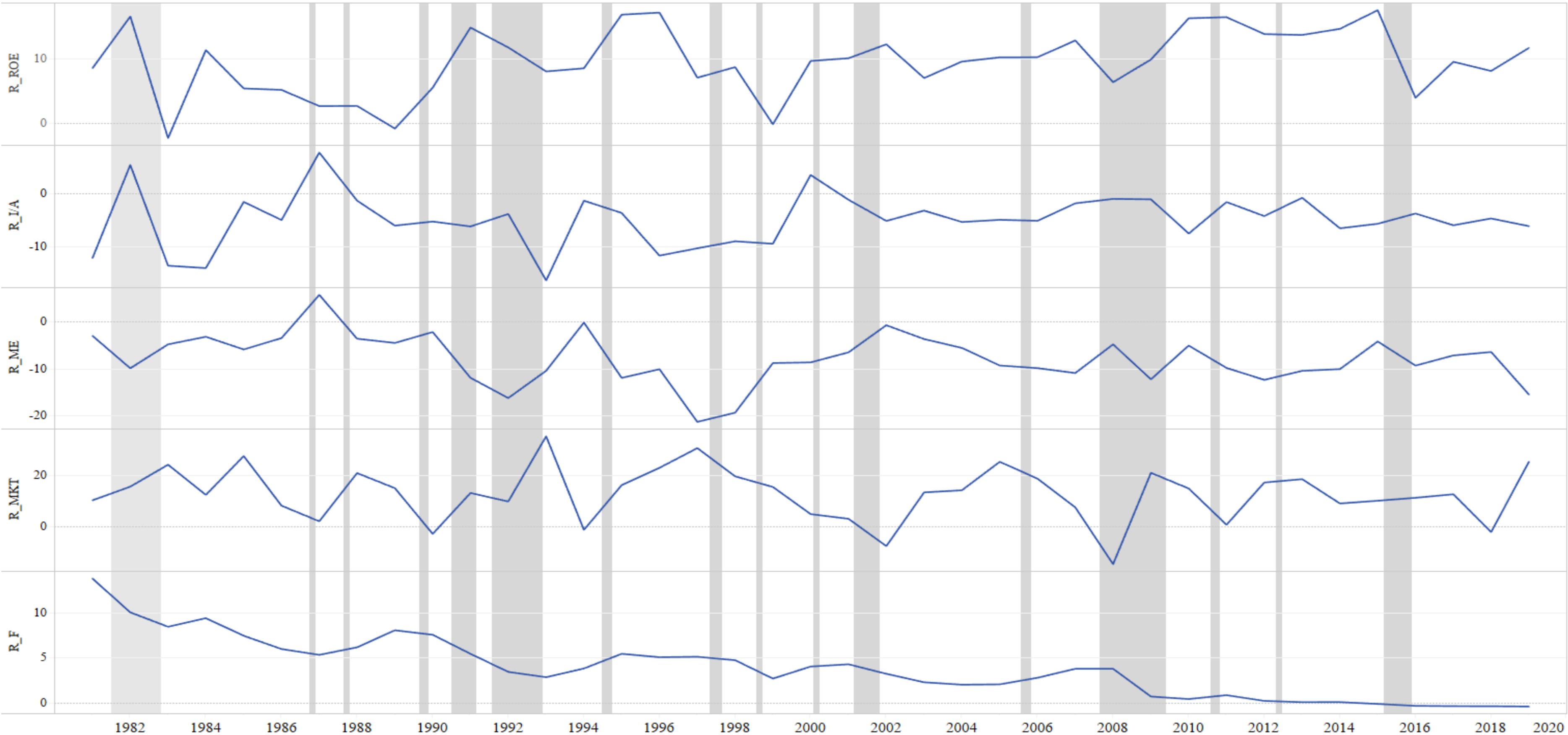


Figure 2. Yearly Q-Factor Premiums of European Equity Market

This Figure plots yearly q-factor returns and the risk-free rate of the European stock market from 1981 to 2019 (in %), 39 years in total. The yearly risk-free rate after 1999 is the annualized EONIA rate, and the yearly rate of one-month U.S. T-bill is used as proxy pre-1999. The yearly q-factor returns are the sum of the monthly returns each year. The market factor MKT is constructed as the value-weighted market return R_{MKT} of the whole sample each month subtract risk-free rate. R_{ME} is the difference between the simple average return of the 9 small size portfolios and the simple average return of 9 big size portfolios. $R_{I/A}$is the difference between the simple average return on the 6 low I/A portfolios and the simple average return 6 high I/A portfolios. And R_{ROE} is the difference between the simple average return of the 6 high ROE portfolios and the simple average return of 6 low ROE portfolio. The grey bands indicate economic financial crisis and NBER recession indicators: 1981.7-1982.11, 1986.10-1986.12, 1987.9-1987.11, 1989.9-1989.12, 1990.7-1991.3, 1991.8-1992.12, 1994.7-1994.10, 1997.5-1997.9, 1998.8-1998.10, 2000.2-2000.4, 2001.3-2001.11, 2005.8-2005.11, 2007.9-2009.6, 2010.8-2010.10, 2012.5-2012.7, 2015.3-2015.12.



will lead to a higher share price. Besides, the investment factor is conditional on a given level of ROE (Asad et al., 2017). If a company does not generate enough profit to cover the operation, investment decisions will be postponed in most cases.

In spite of fluctuations, the annual ROA factor premium remains positive over time except for those recession periods in the early 1980s when the annual return dropped dramatically to around -2%. The profitability effect earns an average monthly return of 0.79% from 1981 to 2019 in Europe, close to that reported by Huber and Preissler (2020) but lower than the premium of 0.84% estimated by Ammann et al. (2012) for 10 countries of the European Monetary Union. The profitability premium of the U.S. market is moderately lower at 0.52%. This indicates that European stocks in general produce more returns attributed to profitability than the stocks in the U.S. market. The difference could also be caused by the survivorship bias, since the U.S. sample of Hou et al. (2015, 2019, 2020) includes all the historical active and dead companies while the European sample of this study mainly consists of active companies. Firms that have been delisted from the exchange are more likely to have poor ROEs, and this in turn will raise the average return of the remaining stocks.

4.6.3 Return Patterns in Europe

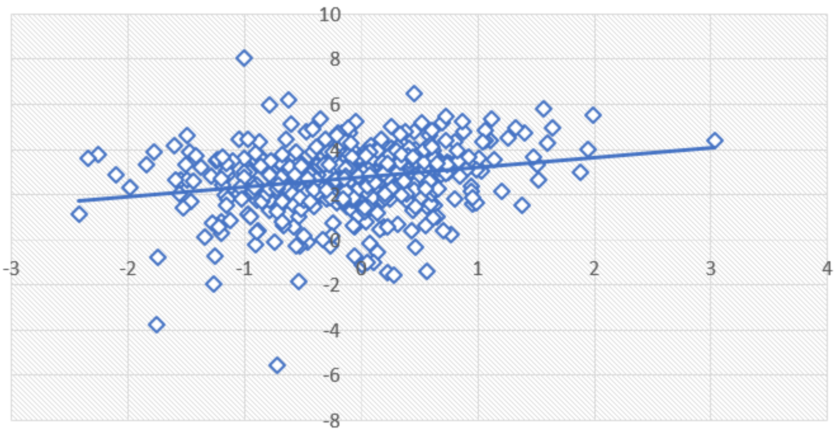
To further elaborate the inference of the return patterns in the European equity market, this section plots scatter diagrams with the time-series coordinates composed of factor return and portfolio return. Portfolios are again grouped as study objects. For instance, to study the size effect, all the small portfolios groups (SLL, SLM, SLH, SML, SMM, SMH, SHL, SHM, SHH) are combined to form a small-sized portfolio and all the big portfolios (BLL, BLM, BLH, BML, BMM, BMH, BHL, BHM, BHH) are combined to form a big-sized portfolio. And the monthly return is calculated as the average of all the component portfolios. For the investment factor, 6 low-investment portfolios (SLL, SLM, SLH, BLL, BLM, BLH) are grouped together, 6 mid-investment portfolios (SML, SMM, SMH, BML, BMM, BMH) are grouped together, and 6 high-investment portfolios (SHL, SHM, SHH, BHL, BHM, BHH) are grouped together. Applying the same logic, the low-profitability portfolio comprises the 6 portfolios with low investment (SLL, SML, SHL, BLL, BML, BHL), the mid-profitability portfolio comprises the 6 portfolios with median investment (SLM, SMM, SHM, BLM, BMM, BHM), and the high-profitability portfolio comprises the 6 portfolios with high investment (SLH, SMH, SHH, BLH, BMH, BHH).

Figure 3 displays the relationship between the portfolio return and the size premium. The slope coefficient is positive for the small-sized portfolios and negative for the big-sized portfolios. With the average size effect being negative at -0.65% per month, the small-sized portfolio earns negative returns attributed to the size factor while the big-sized portfolio earns positive size premiums. This result is the opposite of Fama and French (1993), but as discussed earlier, the size effect is reported divergently in different literature published after 2000.

Figure 3. Relationship between the ME Factor and Portfolio Return

This scatter diagram plots the time-series datasets (in %) of the size variable and portfolio return. The size premium is assigned to the horizontal axis and the portfolio return is assigned to the vertical axis. In Graph A, portfolio return is the average return of 9 small-sized portfolios, each month, from Jan. 1981 to Dec. 2019. In Graph B, portfolio return is the average return of 9 big-sized portfolios, each month, from Jan. 1981 to Dec. 2019.

Graph A. Small-Sized Portfolio



Graph B. Big-Sized Portfolio

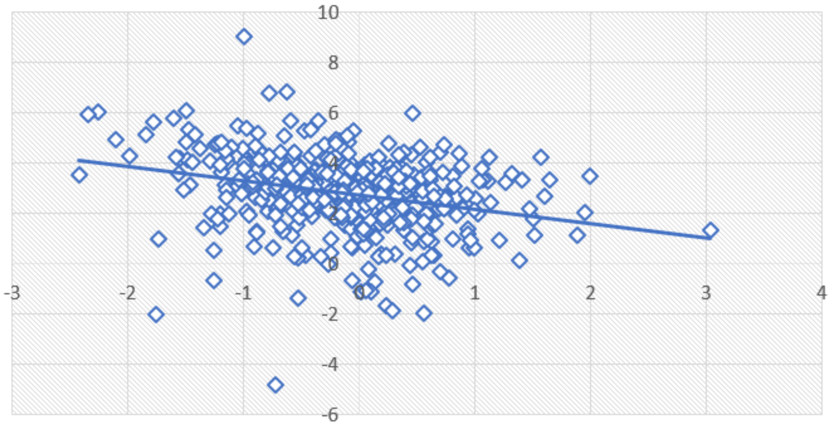
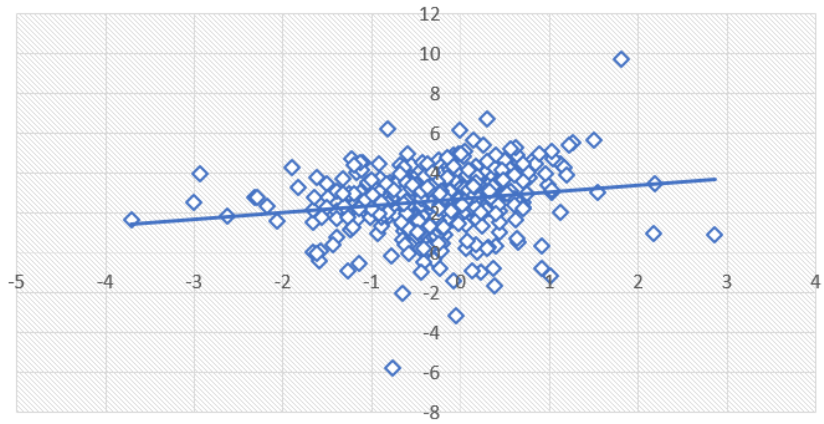


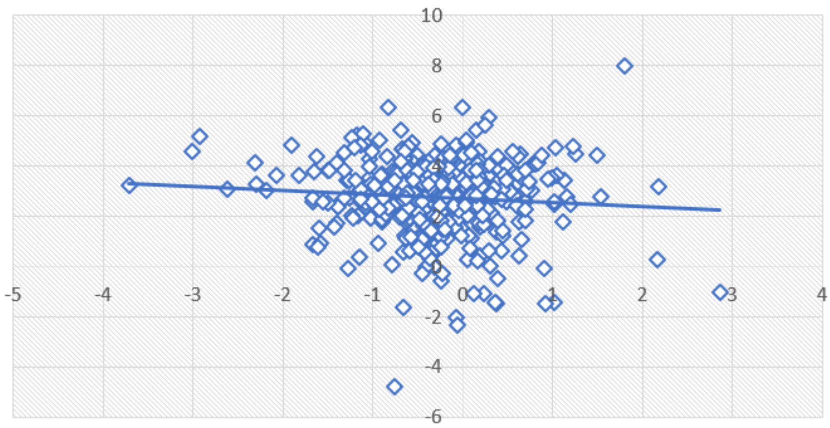
Figure 4. Relationship between the I/A Factor and Portfolio Return

This scatter diagram plots the time-series datasets (in %) of the investment variable and portfolio return. The investment premium is assigned to the horizontal axis and the portfolio return is assigned to the vertical axis. In Graph A, portfolio return is the average return of 6 low-investment company groups, each month, from Jan. 1981 to Dec. 2019. In Graph B, portfolio return is the average return of 6 mid-investment company groups, each month, from Jan. 1981 to Dec. 2019. In Graph C, portfolio return is the average return of 6 high-investment company groups, each month, from Jan. 1981 to Dec. 2019.

Graph A. Low-Investment Portfolio



Graph B. Mid-Investment Portfolio



Graph C. High-Investment Portfolio

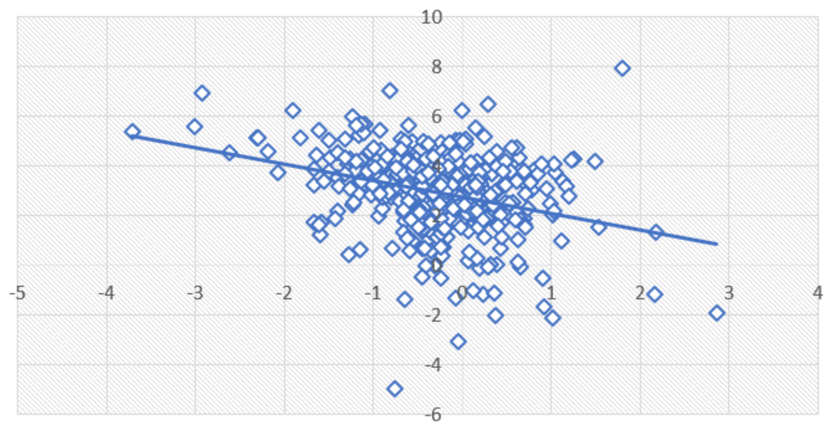
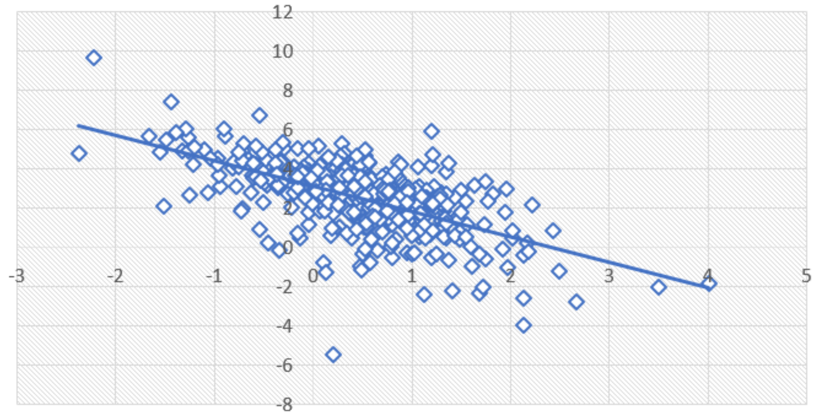


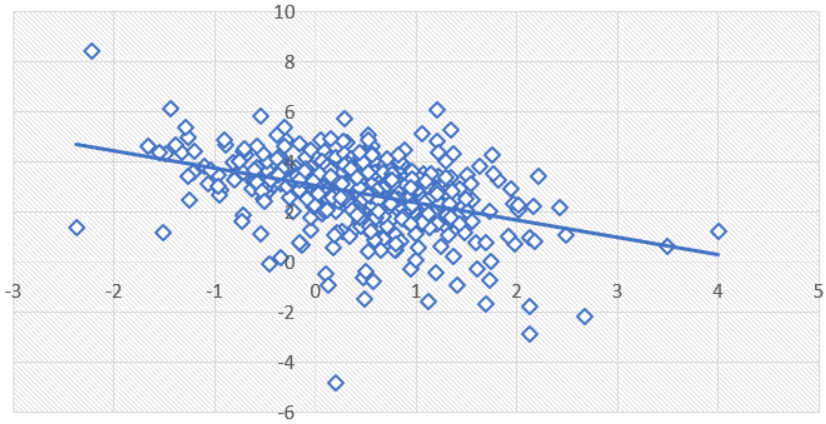
Figure 5. Relationship between the ROE Factor and Portfolio Return

This scatter diagram plots the time-series datasets (in %) of the profitability variable and portfolio return. The ROE premium is assigned to the horizontal axis and the portfolio return is assigned to the vertical axis. In Graph A, portfolio return is the average return of 6 low-profitability company groups, each month, from Jan. 1981 to Dec. 2019. In Graph B, portfolio return is the average return of 6 mid-profitability company groups, each month, from Jan. 1981 to Dec. 2019. In Graph C, portfolio return is the average return of 6 high- profitability company groups, each month, from Jan. 1981 to Dec. 2019.

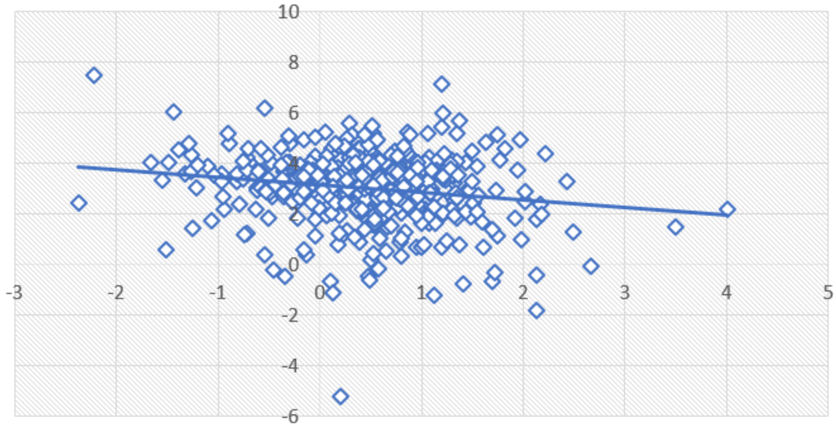
Graph A. Low-Profitability Portfolio



Graph B. Mid-Profitability Portfolio



Graph C. High-Profitability Portfolio



In Figure 4, the scatterplots of portfolio returns and the investment factor premiums are presented. The low-investment portfolio has a positive correlation with I/A factor, and portfolios with median investment and high investment have a negative correlation with I/A. The risk exposure of the high-investment portfolio is stronger than the mid-investment portfolio. Given a negative average monthly I/A premium of -0.41%, the return generated by the investment factor increases conditionally as the investment becomes aggressive. As such, high-investment stocks produce higher returns than low-investment stocks in Europe. This result is aligned with Walkshäusl and Lobe (2014) but contradicting with most of the prior research (e.g., Ammann et al., 2012; Barillas et al., 2019; Fama and French, 2015; Flechter, 2019; Hou et al., 2015; Huber and Preissler, 2020) as well as the capital budgeting principles which demonstrate that low return indicates high investment as the NPV will be higher with a lower discount rate.

The relationship between portfolio returns and the ROE premiums is illustrated in Figure 5. The slope coefficients of all three profitability portfolios are negative. As the profitability increases from low to high, the slope magnitude decreases. Under a positive average profitability premium, the return of high-profitability stocks is higher than that of low-profitability stocks. This inference is consistent with almost all of the existing related research.

To sum up, these scatter diagrams rationalize the rough pattern of European stock return mentioned in the earlier section: stocks of big companies generate higher returns than those of small companies, stocks of high-investment companies have higher returns than those of low-investment companies, and stocks of high-profitability companies generate higher returns than those of low-profitability companies.

4.7 Validity Testing

4.7.1 CMA Factor Construction

Firstly, in order to double verify the unexpected result of the I/A premium, CMA (conservative-minus-aggressive), the investment factor of the Fama-French 5-factor pricing model is constructed and computed. Following the procedure of Fama and French (2015), CMA portfolio groups are constructed at a European level based on a 2-by-3 sort. Each month, stocks are first assigned to two size groups, small (S) and big (B), by comparing with the median market capitalization, and then split into three groups, conservative (C), neutral (N), and aggressive (A), according to the breakpoint of 30th and 70th percentile of I/A, The intersection of the blocks creates 6 portfolios that are labeled as SC, SN, SA, BC, BN, and BA. The CMA is defined as the difference between the average return on diversified portfolios of conservative-investment stocks and aggressive-investment stocks: $CMA = (SC + BC)/2 - (SA + BA)/2$.

Table 6 exhibits the average value-weighted returns of 6 size-investment portfolios and the CMA factor return. As presented in Panel A, SC portfolio earns the lowest monthly return (0.01%) and the BA portfolio earns the highest return (1.27%) among the 6 stock groups. Portfolios which consist of small-sized companies tend to have lower returns than those consist of big-sized companies. And portfolios of aggressive-investment companies tend to have higher returns than those of conservative-investment companies. The pattern observed here is exactly the same as the I/A factor. Panel B shows that the monthly average premium of the CMA factor is also negative with a mean value of -0.59%, even lower than the I/A factor (-0.41%).

Table 6. Summary Statistics of CMA Portfolio Groups

Panel A: Summary Statistics of Portfolio Return from Jan. 1981 to Dec. 2019, 468 months.
This panel presents the summary statistics of 6 size-investment portfolios over the period from January 1981 to December 2019, including mean value-weighted return (in %), standard deviation, maximum value, and minimum value.

Portfolio	Obs.	Mean	SD	Min	Max
SA	468	0.763031	2.048201	-6.242935	6.462900
SN	468	0.475772	1.842125	-5.674676	5.484955
SC	468	0.013249	1.929765	-6.882343	5.583724
BA	468	1.273452	2.835425	-8.593190	7.698651
BN	468	1.094017	2.784605	-9.125885	7.640752
BC	468	0.837636	2.758495	-7.390667	6.827839

Panel B: Time-Series Factor Premium
CMA is the difference between the simple average return on the 2 low I/A portfolios and the simple average return 2 high I/A portfolios.

Return Factor	Mean	SE	T-value	Median	SD
CMA	-0.592799	0.061005	9.717191	-0.669133	1.319744

The investment premium inconsistency in Europe and the U.S. might be caused by different data sources. The company master data and accounting information of this research are retrieved from Thomson Reuters Datastream while Hou et al. (2015, 2020) and some other research apply the CRSP as the main data source. The data from these two platforms differentiate in the coverage, classification, treatment of corporation actions, and the reporting of data for inactive firms (Ice and Porter, 2006).

The difference in data coverage and classification errors will lead to different breakpoints and the market return calculation. Furthermore, the survivorship bias could also contribute partially to this negative investment premium. During the construction process of the integrated European index, delisted companies are excluded from the sub-indices and are not added back due to the data unavailability in Thomson Reuters Datastream.

4.7.2 Cross-sectional Regression Testing

To evaluate the validity of the four q-factor return in Europe, this thesis applies the factor regression methodology introduced by Fama and MacBeth (1973) which serves as a common tool in recent empirical works in terms of asset pricing (Fama and French, 2020). Fama and French (1973, 1993) run the cross-sectional regression testing over all the individual sample stocks to estimate the factor loadings more precisely. However, evidence from the efficiency study of adopting individual stocks and portfolios in tests of cross-sectional asset pricing models (Ang et al., 2020) argues that there is no difference estimating factor risk premia using the Fama-French approach or using portfolios as test assets. They report the same risk premium coefficients by running the cross-sectional regression using individual stocks and portfolios. Thus, instead of running over all stocks which would be technically redundant, the 24 portfolio groups (18 size-investment-profitability portfolios and 6 size-investment portfolios) are used as testing assets for the factor regression in this thesis.

The q-factor model can be viewed as the below constrained multivariate linear expression which regresses the excess return on systematic factors:

$$R_{i,t} - R_f = \alpha_{i,t} + \beta^i_{MKT} R_{MKT,t} + \beta^i_{ME} R_{ME,t} + \beta^i_{I/A} R_{I/A,t} + \beta^i_{ROE} R_{ROE,t} + \epsilon_{i,t} \quad (19)$$

where $R_{i,t}$ denotes the monthly return of portfolio group i in month t, R_f denotes the risk-free rate, $R_{i,t} - R_f$ is the monthly excess return, β^i_{MKT}, β^i_{ME}, $\beta^i_{I/A}$, and β^i_{ROE} are the respective factor loadings of market premium $R_{MKT,t}$, size premium $R_{ME,t}$, investment premium $R_{I/A,t}$, and profitability premium $R_{ROE,t}$. And $\alpha_{i,t}$ represents the intercept, $\epsilon_{i,t}$ denotes the error term of the linear regression.

A multicollinearity test is performed before the regression process to ensure that there is no high correlation among the four variables. If two or more variables are highly correlated, the estimate of the coefficients could be problematic and inaccurate. As presented in the correlation matrix in Table 7, the correlation among the four q-factors is very weak, lying between -0.17 to 0.09.

Table 7. Factor Correlation Matrix

	R_F	MKT	R_ME	R_I/A	R_ROE	CMA
R_F	1	-0.600629	0.019805	0.056280	0.021328	0.062347
R_MKT	-0.600629	1	-0.174522	-0.123679	-0.030532	-0.120548
R_ME	0.019805	-0.174522	1	0.017653	-0.015923	0.027663
R_I/A	0.056280	-0.123679	0.017653	1	0.088649	0.941323
R_ROE	0.021328	-0.030532	-0.015923	0.088649	1	-0.096306
CMA	0.062347	-0.120548	0.027663	0.941323	-0.096306	1

The testing result and summary statistics of the cross-sectional regression on 24 portfolios are provided in Table 8. The market β_{MKT} is significant and positive for all the portfolios, fluctuating around 1. Varying from 0.72 to 1.16, the size β_{ME} of small-sized groups are statistically significant while half of the big-sized portfolios have insignificant slope coefficients. The big-sized portfolios are less sensitive to the size effect compared to small-sized portfolios with betas lying between -0.19 to 0.12. Given the negative size factor premium, big portfolios earn higher returns than small portfolios.

The investment beta ranges from -0.61 to 0.62 and is significant for 22 out of 24 portfolios. Low-investment portfolios have positive factor loadings while high-investment portfolios have negative factor loadings. $\beta_{I/A}$ decreases when the investment strategy becomes more aggressive. The average beta of 8 low-investment groups stands at 0.53 whereas for 8 high-investment groups, the mean of beta coefficient is only -0.47. Since the investment factor I/A is negative, low-investment portfolios have higher returns attributed to the investment factor than high-investment portfolios.

The ROE factor is significant for all the majority sample groups except three medium-profitability portfolios (SHM, BLM, and BHM) and two medium-investment portfolios (SN and BN). As the ROE increases, β_{ROE} augments accordingly. ROE factor loadings of high-profitability portfolios are larger than the low-profitability portfolios. Slope coefficients are positive for all high-profitability stock samples (0.40

on average) and are negative for all low-profitability stock samples (-0.61 on average), indicating a positive relationship between the profitability of a company and the stock return. This finding is in line with existing research (e.g., Fama and French, 2015; Hou et al., 2015, 2020; Barillas et al., 2019; Huber and Preissler, 2020).

The above testing results on the size factor, the investment factor, and the profitability factor are largely consistent with the observed return pattern in Europe discussed in earlier sections. 10 out of 24 portfolios have statistically insignificant alpha values, and the intercepts of the other groups are reasonably small and close to 0. As shown in Panel 2, the average intercept value of all the sample groups is around 0.0007. Most of the beta coefficients of the four q-factors are significant with t-values larger than 2. And the adjusted R-square ranges from 80% to 97%, implying a high degree that the four q-factors explain the dependent variable in this linear regression model. As such, it is safe to conclude that the q-factor model is well specified and has excellent explanatory power for the excess return of the European stock market.

Table 8. Cross-Sectional Factor Regression on 24 Portfolios

Panel A presents the testing result of cross-sectional q-factor model regression on 24 portfolio groups (18 size-investment-profitability portfolios and 6 size-investment portfolios). The monthly excess return $R_{i,t} - R_f$ is the dependent variable. The market factor return MKT, size factor return $R_{ME,t}$, investment factor return $R_{I/A,t}$, and profitability factor return $R_{ROE,t}$ are taken as independent variables. α is the intercept and should be economically small and statistically insignificant if the q-factors well explain the excess return of the portfolio. Panel B shows the summary statistics of the intercept and beta coefficients. T-statistics are indicated in the parentheses.

Panel A: Testing Result of Cross-sectional Q-Factor Model Regression on 24 Portfolio Groups

	SLL	SLM	SLH	SML	SMM	SMH
α	-0.0050	0.0004	0.0041	-0.0035	0.0004	0.0027
	(-5.0403)	(0.3641)	(3.1359)	(-2.7793)	(0.5256)	(2.8481)
β_MKT	1.0425	0.9662	1.0752	1.0164	0.8747	1.0056
	(65.4005)	(61.2222)	(51.1272)	(50.9963)	(65.8646)	(67.0091)
β_ME	1.0427	0.8684	1.1035	1.1099	0.7249	0.9368
	(28.4402)	(23.9267)	(22.8167)	(24.2152)	(23.7352)	(27.1428)
β_I/A	0.4322	0.4821	0.5247	0.0214	0.0819	0.0535
	(11.8968)	(13.4058)	(10.9495)	(0.4712)	(2.7077)	(1.5632)
β_ROE	-0.5979	-0.1021	0.4141	-0.5386	-0.1143	0.3207
	(-17.2068)	(-2.9666)	(9.0341)	(-12.3978)	(-3.95)	(9.8023)
Adj.R^2	92.97%	90.53%	85.75%	89.06%	91.73%	91.13%
Obs.	468	468	468	468	468	468

	SHL	SHM	SHH	BLL	BLM	BLH
α	0.0009	-0.0007	0.0075	0.0052	0.004	-0.0019
	(0.6083)	(-0.6443)	(8.0844)	(4.4671)	(3.7402)	(-1.3694)
β_MKT	1.0194	1.0084	1.021	1.0299	1.0277	0.9415
	(40.9982)	(60.3273)	(69.4218)	(55.8784)	(60.6243)	(41.6642)

β_ME	1.1616	0.9775	0.9324	0.1162	-0.0753	0.0507
	(20.3119)	(25.4277)	(27.5667)	(2.7411)	(-1.9309)	(0.9756)
β_I/A	-0.6074	-0.4071	-0.3477	0.6224	0.5273	0.588
	(-10.7203)	(-10.6882)	(-10.3761)	(14.8194)	(13.6503)	(11.4192)
β_ROE	-0.4991	-0.0235	0.3244	-0.6345	-0.0292	0.4878
	(-9.2098)	(-0.6442)	(10.1189)	(-15.7938)	(-0.7903)	(9.903)
Adj.R^2	85.25%	90.77%	92.09%	90.65%	90.52%	80.35%
Obs.	468	468	468	468	468	468
	BML	BMM	BMH	BHL	BHM	BHH
α	0.0057	-0.00001	-0.0049	0.0021	-0.0013	-0.0019
	(4.4595)	(-0.0847)	(-4.5953)	(1.4183)	(-1.094)	(-1.7988)
β_MKT	1.041	0.9659	0.9891	0.9662	0.985	1.0831
	(50.9325)	(69.7874)	(57.7263)	(40.2906)	(53.6209)	(62.8074)
β_ME	-0.1978	-0.1076	0.0368	-0.056	-0.0253	0.1162
	(-4.2088)	(-3.3805)	(0.935)	(-1.0161)	(-0.5998)	(2.929)
β_I/A	0.0762	0.0641	-0.1837	-0.4945	-0.3818	-0.5845
	(1.6354)	(2.0339)	(-4.7046)	(-9.0492)	(-9.1218)	(-14.8751)
β_ROE	-0.7119	-0.0617	0.3967	-0.6627	-0.0126	0.4118
	(-15.9788)	(-2.0446)	(10.6216)	(-12.6776)	(-0.3147)	(10.9545)
Adj.R^2	89.68%	92.81%	88.87%	85.32%	88.74%	90.97%
Obs.	468	468	468	468	468	468
	SA	SN	SC	BA	BN	BC
α	0.0033	0.0007	-0.0011	-0.0011	-0.0004	0.0024
	(5.9486)	(1.247)	(-2.0069)	(-1.821)	(-0.7402)	(3.9347)
β_MKT	1.0135	0.9401	1.0269	1.0232	0.9829	1.005
	(114.7638)	(99.8471)	(116.159)	(109.5823)	(115.2513)	(102.1432)
β_ME	0.9774	0.8497	0.9914	0.049	-0.0809	-0.0064
	(48.1236)	(39.2398)	(48.7644)	(2.282)	(-4.126)	(-0.2836)
β_I/A	-0.4399	0.0696	0.4919	-0.4847	-0.0179	0.5862
	(-21.8586)	(3.2455)	(24.4206)	(-22.7817)	(-0.9205)	(26.1454)
β_ROE	0.0542	-0.0127	-0.2315	0.0931	-0.0264	-0.0654
	(2.818)	(-0.6179)	(-12.0124)	(4.5748)	(-1.4223)	(-3.0516)
Adj.R^2	97.22%	96.12%	97.31%	96.99%	97.21%	96.45%
Obs.	468	468	468	468	468	468
Panel B: Summary Statistics of Intercept and Beta Coefficient						
	Mean	SE	Median	SD	Min	Max
α	0.000729	0.000658	0.000400	0.003224	-0.005000	0.000729
β_MKT	1.002100	0.009233	1.010950	0.045231	0.874700	1.002100
β_ME	0.478992	0.105312	0.420550	0.515922	-0.197800	0.478992
β_I/A	0.028013	0.086678	0.058800	0.424636	-0.607400	0.028013
β_ROE	-0.075888	0.075550	-0.027800	0.370116	-0.711900	-0.075888

5. The Application of Q-Factor Returns in European PE fund Benchmarking

5.1 Data Source and Dataset Specification

Driessen et al. (2012) obtain quarterly private equity cash flows and NAVs from the TVE. The data quality of the TVE is frequently criticized by recent researchers. Higson and Rüdiger (2012) identify missing cash flows from the database and indicate that the dataset is not timely updated in this platform, leaving a large amount of value, including NAVs, outdated or unchanged for many periods. Harris et al. (2014) suggest that the PE fund performance based on the TVE samples is significantly downward biased. Preqin, on the other hand, has a better data coverage for alternative assets. The cash flow and NAV data in Preqin are updated in a timely manner, especially from 2000 onwards. Besides, compared to other widely used alternative asset financial data platforms such as the Burgiss, and the CA (Cambridge Association), the Preqin dataset is more reliable and has a less chance of misreporting due to the availability of more diversified sources (i.e., LP fund valuation reports, GP valuation reports, Pension fund reports, etc.), which can be used for cross-checking and verification (Phalippou, 2014). Based on the experience of former research, the Preqin database is used in this study to analyze the European private equity fund performance given its completeness and credibility.

European private equity data including fund name, fund size (i.e., commitment), asset class, vintage year (inception date), cash flow transactions, and the NAVs are exported from Preqin on a quarterly basis. The sample runs from September 1985 to December 2019 and contains buyout funds and venture capital. Transactions data consist of capital calls, distributions, and net cash flows on the individual fund level. According to the data specification in Preqin, all the cash flows are scaled to a 10 million commitment basis, which means cash flows are not recorded based on the original fund size. In the case that the fund size information is not available, the median commitment of the sample is set as the missing value. Therefore, the raw transaction data are all equal-weighted. And this will probably neutralize the effect generated by small-sized funds and big-sized funds. To get the value-weighted cash flows, it requires every transaction to multiply by a scaling ratio (i.e., original fund size divided by 10 million).

One defect of the dataset is that the cash flow transactions are denominated in local currency while the fund size currencies are unified, and the local currency of each particular fund is not specified in the raw data. As such, there is no way either to identify the corresponding foreign exchange rate or to convert the local values into Euro. Therefore, a necessary assumption is required that the local currency of all these European private equity funds is Euro. As a manner of fact, Euro is the major currency of the European region focused funds; thus, the empirical impact of this currency issue is negligible.

5.2 Descriptive Statistics of Private Equity Data

As presented in the descriptive statistics in Table 9, the European private equity sample of this study has a population of 588 funds, including 517 buyout funds and 71 venture capital funds. Buyout funds occupy more than 87% of the whole samples, indicating that in the past few decades, buyout funds play a dominant role in the private equity investment. This is mainly because venture capital is more risk-taking as it invests in early-stage startups whilst the underlying targets of buyout funds are normally mature companies.

The average fund size of the buyout fund stands at 1532.9 million Euros, more than 8 times that of the venture capital (182.5 million). Funds with a commitment of less than 5 million are removed from the database in some literature (Driessen et al., 2012, Phallipou and Gottschalg, 2009). Since the smallest commitment size is 6.18 million for venture capital and 14.26 million in this sample, it is not necessary to run this filter. There are 14876 cash flow transactions for buyout funds and 1613 transactions for venture capital funds. All the cash flows are calculated with a value weighting approach, scaling to the original fund size. The NAVs of the whole sample add up to 319.5 billion, of which 312.4 billion comes from buyout funds and 7.0 billion are from venture capital.

The average contribution and distribution of the buyout fund are 1216.6 million and 1294.3 million, respectively. The average transaction amounts of venture capital funds are much smaller, with a mean contribution of 145.4 million and a mean distribution of 122.3 million. Total capital contributions of European PE funds amount to 639.3 billion and total distributions amount to 677.8 billion; TVPI and DPI of all of the funds are 1.56x and 1.06x respectively. Buyout funds have a slightly better TVPI and a considerably higher DPI than venture capital. This is mainly because the venture capital funds are incepted later and most of them still have a big residual value. According to these traditional multiple indicators, European buyout funds outperform European venture capital funds to a small degree.

Table 9. Descriptive Statistics of European Private Equity Funds

This table gives the descriptive statistics of the European private equity fund sample, including fund number, fund size, NAV, contribution, distribution, net cash flow, TVPI, DPI, and fund effective years. Data span from September 1985 to December 2019. Fund size is the total commitment of capital from investors. *Mean fund size* calculation excludes those funds whose commitment information is missing in the raw data output from Preqin. NAV denotes the residual value that is not yet realized, and *sum NAV* is the residual value of all the funds. *Sum Contribution* denotes the cumulative capital investment of all the funds and *Sum Distribution* denotes the cumulative return from the investment of all the funds. Net cash flow is the difference between total cumulative contribution and total cumulative distribution. Mean values are equal to the respective sum values divided by the fund numbers. TVPI is calculated as the sum of total NAV and total cumulative distribution divided by total cumulative contribution and DPI is computed as *Sum Distribution* divided by *Sum Contribution*. All cash flow transactions and NAVs are converted with a value weighting approach to the original fund size. *Fund Effective Years* is the fund duration which starts from the inception date to the final cash flow date.

	BO	VC	All Funds
No. of Funds	517	71	588
No. of Liquidated Funds	67	10	77
No. of Non-liquidated Funds	450	61	511
Mean Fund Size (€ MN)	1532.9	182.5	1362.9
Sum NAV (€ BN)	312.4	7.0	319.5
Mean NAV (€ MN)	604.3	99.2	543.3
Sum Contribution (€ BN)	629.0	10.3	639.3
Mean Contribution (€ MN)	1216.6	145.4	1087.2
Sum Distribution (€ BN)	669.2	8.7	677.8
Mean Distribution (€ MN)	1294.3	122.3	1152.8
Sum Net Cash Flow (€ BN)	40.2	-1.6	38.6
Mean Net Cash Flow (€ MN)	77.8	-23.2	65.6
No. of Cash Flows	14876	1613	16489
TVPI	1.56	1.52	1.56
DPI	1.06	0.84	1.06
Fund Effective Years	9.5	10.2	9.99

5.3 Discussion on the Non-liquidated Fund Issue and NAV Treatment

As reported in Table 9, only 67 buyout funds and 10 venture capital funds out of 588 funds are liquidated; the rest are non-liquidated funds. Since the non-liquidated funds can significantly affect the return estimation with their incomplete cash flows and larger NAVs, the performance is constantly better in liquidated fund observation groups than the full sample (Korteweg and Nagel, 2016). In some previous studies (e.g., Farrelly and Stevenson 2019; Buchner, 2014; Franzoni et al., 2012), only fully liquidated funds are included in the sample to test systematic risk and abnormal return of private equity investments. Some research adds additional criteria on the non-liquidated fund sample to proxy for liquidation, such as returns are unchanged for at least the final six quarters (Kaplan and Schoar, 2005), or funds are at least 7 to 10 years old (Driessen et al., 2012; Phalippou and Gottschlag, 2009; Ang et al., 2018). There are also different voices from Robinson and Sensoy (2016), they argue that performance assessments are not sensitive to the inclusion of non-liquidated assets.

In this study, the original sample population is much smaller than other research work. Non-liquidated funds account for more than 85% of the total sample and the inception dates of most funds are in post-2005. Thus, removing all the non-liquidated funds or only selecting the old funds will cause severe sample bias. Considering the average of fund effective years already reaches around 10 years even with the non-liquidated funds, in order to remain a reasonable sample size, all the non-liquidated funds are included and no further data screening towards this sample is performed.

Another issue that needs to be discussed is the residual NAV of non-liquidated funds, which has always been an issue for the risk and return estimations because different treatment of the NAV will lead to considerably different empirical results. Treating the NAV as the liquidating cash inflow at the end of the sample period is the most widely used method (Kaplan and Schoar, 2005; Korteweg and Nagel, 2016; Robinsen and Sensoy, 2016). However, the self-reported NAV valuation can be easily manipulated by fund managers. Brown, Gredil, and Kaplan (2019) test the presence of return manipulation with a large dataset of buyout funds and venture capital and find that GPs appear to maintain conservative valuation for top-quartile funds while inflating the return for underperforming funds, especially during fundraising. Kleymenova, Talmor, and Vasvari, (2012) suggest substituting the NAV with the market price in the secondary market which approximately equals 75% of the NAV. This is challenged by Phalippou (2014) as he finds that the market value is 25% higher than the stated NAV. Another assumption that applied by Phalippou and Gottschlag (2009) is to directly write off the NAV. Under this circumstance, the performance of PE investment, which is measured by a profitability index, decreased dramatically by 7%. Driessen et al. (2012) come up with a compromised methodology by converting the NAV into a market value that adjusted for systematic risk.

Nevertheless, to identify the actual market value requires massive data computation. Given the primary purpose of PE benchmarking analysis in this research is to serve as an example to testing the q-factor returns, this study follows the mainstream and applies the straightforward approach to treat the final NAV as cash inflow of the last sample period.

5.4 Fund of Fund Portfolio Construction

To use the GMM estimation framework elaborated in chapter three, it is required to have many different vintage years and funds from each different vintage year. This research adopts a classic approach from some prior PE studies (Buchner, 2014; Driessen et al., 2012; Franzoni et al., 2012) to form fund-of-funds (FoFs), more specifically, to group individual PE investment into portfolios based on vintage year or investment starting date. In this way, the effect of idiosyncratic shocks can be greatly reduced. Additionally, this methodology will also contribute to a more sufficient dispersion in the explanatory variables (Franzoni et al., 2012) and consequently endow the risk premia estimation with greater statistical power (Buchner, 2014).

Private equity funds incepted in the same year are grouped together to form an FoF. FoFs are constructed separately for the venture capital segment and the buyout segment. All cash flow transactions are converted to the original amount by multiplying the scaling factor. The net cash flow of each FoF is aggregated by merging the cash flows including NAV of all the individual investments each year. The IRR of each FoF is computed based on the cash flow streams. The result of the fund formation and the aggregated net annual cash flow of each FoF are presented in Table 10.

23 venture capital FoFs and 28 buyout FoFs are constructed in total. As can be seen in Panel A and Panel B, venture capital investment starts earlier than buyout investment, but it does not perk up until post-2000. The investment grows rapidly after 2001 and the IRR increases dramatically from 2% to 21% in 6 years. Due to the financial crisis in 2007-2009, IRR reaches its lowest point. Investment drops substantially in 2009 and continues the raising path starts in 2010 following the modest economic recovery. The IRR of venture capital funds fluctuates from 8% to 22% afterwards. For buyout funds, the investment demonstrates a similar tendency as venture capital post-crisis and the IRR remains reasonably stable from 11% to 18% over the past decade. The highest IRR lies in the middle 90s and the beginning of 21 century. With a higher average IRR at 18%, buyout funds seem to have better performance than venture capital funds (14%). This is in line with the performance indicated by TVPI and DPI measures.

Table 10. Aggregated Cash Flow of FoF Portfolios

Panel A: Net Cash Flow of Venture Capital FoFs: in Millions of Euros

This Panel presents the aggregated cash flow of each fund-of-fund in millions of Euros. All the venture capital funds with the same vintage year are grouped together to form an FoF. The years missing from 1985 to 2019 in this table indicate that there is no venture capital fund incepted from those years, hence no FoF is constructed. All cash flow transactions are scaled to the original amount based on the fund size. FoF cash flows are computed by aggregating all cash flow of each underlying fund each year.

	Vintage Year																						
Year	1985	1990	1991	1996	2000	2001	2002	2003	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
pre1990	-118	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1990	-56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1991	1	-26	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1992	10	-18	-17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1993	11	-46	-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1994	47	-12	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1995	69	-3	74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1996	29	-17	62	-33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1997	39	62	43	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1998	30	38	24	-39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1999	5	74	40	-21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2000	72	63	74	70	-87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2001	0	13	2	11	-21	-34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2002	0	17	4	-1	-24	-138	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2003	0	31	8	7	-14	-87	-94	-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2004	0	0	12	81	-45	-94	-52	-54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	16	0	10	-48	89	-104	-70	-65	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2006	0	62	0	30	-17	-34	-49	-34	-197	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2007	0	0	0	5	-24	55	48	-63	-107	-54	-164	0	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	0	-29	25	97	19	-29	-73	-57	-116	-24	0	0	0	0	0	0	0	0	0	0	0
2009	0	0	-1	0	0	47	69	10	-85	-24	-137	1	-32	0	0	0	0	0	0	0	0	0	0
2010	0	0	0	0	0	40	1	10	-40	-47	-113	-135	-31	-29	0	0	0	0	0	0	0	0	0
2011	0	0	12	0	33	20	18	33	-27	-9	-51	-26	-64	-42	-5	0	0	0	0	0	0	0	0
2012	0	0	0	0	44	59	0	-11	37	-62	123	-81	-43	-29	-67	-30	0	0	0	0	0	0	0
2013	0	0	0	10	153	59	1	19	67	23	-27	57	-2	-25	-15	-121	-26	0	0	0	0	0	0
2014	0	0	0	0	20	5	302	25	311	33	345	-4	80	-20	-74	-97	-69	-34	0	0	0	0	0
2015	0	0	0	1	1	13	7	186	251	123	28	-21	0	83	-50	-67	-61	-74	-74	0	0	0	0
2016	0	0	0	2	62	55	6	47	1	692	164	38	0	4	-28	-40	-88	-69	-111	-195	0	0	0
2017	0	0	0	0	0	1	10	7	99	5	101	7	61	26	188	-16	-61	-71	-40	-117	-412	0	0
2018	0	0	0	0	0	35	0	0	255	42	45	32	9	85	290	-38	127	126	22	-142	-214	-57	0
2019	0	0	0	0	0	0	2	18	166	31	248	251	141	160	65	693	280	181	310	698	1328	42	-17
IRR	6%	17%	33%	13%	2%	6%	6%	3%	8%	21%	8%	4%	9%	17%	22%	12%	9%	8%	13%	22%	NA	NA	NA

Panel B: Net Cash Flow of Buyout FoFs: in Billions of Euros

This Panel presents the aggregated net cashflow of each fund-of-fund in billions of Euros. All the buyout funds with the same vintage year are grouped together to form an FoF. The years missing from 1985 to 2019 in this table indicate that there is no buyout fund incepted from those years, hence no FoF is constructed. All cash flow transactions are scaled to the original amount based on the fund size. FoF cash flows are computed by aggregating all cash flow of each underlying fund each year.

	Vintage Year																											
Year	1990	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
1990	-0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1991	-0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1992	-0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1993	-0.2	-0.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1994	0	-0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1995	0.4	-0.1	-0.3	-0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1996	0.3	-0.7	-0.1	0	-0.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1997	0.3	-0.1	0.4	-0.3	-0.4	-0.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1998	0.6	0.1	0.1	0.1	0	-0.9	-1.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1999	0	0.5	0.1	0.4	0.2	-0.3	-3.0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2000	0.1	0.3	0.1	0.3	0.2	0.2	-2.6	-2.2	-0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2001	0	0.3	0.4	0.2	0.4	0.1	-0.7	-0.8	-1.6	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2002	0	0.2	0.2	0	0.2	0.1	1.4	-1.3	-0.5	-1.0	-1.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2003	0	0.1	0.1	0	0.1	0.2	1.9	0	-0.9	-1.7	-2.6	0.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2004	0	0.2	0.1	0	0.4	0.4	3.1	0.5	0.1	-0.5	-0.9	-0.8	-1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0.4	0.5	4.4	1.1	1.0	1.2	0.7	-0.8	-1.0	-5.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2006	0	0.3	0	0.2	0	0.2	2.2	3.8	2.5	4.6	2.6	-0.1	-2.4	-7.9	-6.2	0	0	0	0	0	0	0	0	0	0	0	0	0
2007	0	0.1	0	0	0	0.2	3.3	3.3	2.8	5.2	7.5	0.3	1.8	-4.6	-19	-13	0	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	0	0	0	0.1	0.1	0.5	0.5	0.8	1.7	0.0	3.0	-6.9	-14	-15	-8.6	0	0	0	0	0	0	0	0	0	0	0
2009	0	0	0	0.1	0	0	0.1	0.3	0.1	0.2	0.0	0.0	0.2	-2.2	-5.0	-4.9	-5.7	-2.2	0	0	0	0	0	0	0	0	0	0
2010	0	0	0	0	0	0.2	0.0	0.5	2.5	0.7	1.2	0.1	-0.2	1.0	-4.8	-8.5	-13	-3.9	-1.9	0	0	0	0	0	0	0	0	0
2011	0	0	0	0	0	0	0.4	1.3	1.1	1.1	1.9	2.3	0.2	6.6	4.1	-3.6	-9.0	-3.9	-2.6	-2.7	0	0	0	0	0	0	0	0
2012	0	0	0	0	0	0	0.2	0.7	0.5	0.4	0.7	0.9	1.0	7.2	6.5	-1.3	-5.7	-1.0	-1.8	-8.9	-3.9	0	0	0	0	0	0	0
2013	0	0	0	0.1	0	0	0.1	0.1	0.2	1.3	0.8	0.8	0.9	9.9	11.7	3.7	4.4	-0.2	-1.2	-4.9	-7.7	-2.2	0	0	0	0	0	0
2014	0	0	0	0	0	0	0.5	0.1	0.1	0.2	1.4	0.6	1.1	7.5	13.3	10.6	6.7	2.2	0.9	-5.9	-7.2	-6.0	-7.0	0	0	0	0	0
2015	0	0	0	0	0	0	0.2	0.1	0.1	0.3	0.7	0.1	1.2	6.9	11.6	19.6	23.1	3.0	1.3	0.9	-8.1	-2.4	-11	-3.0	0	0	0	0
2016	0	0	0	0	0	0	0.1	0.0	0.0	0.2	0.7	0.1	0.7	6.6	15.1	11.0	11.2	3.2	0.7	2.9	-0.8	-1.5	-7.3	-8.3	-3.5	0	0	0
2017	0	0	0	0	0	0	0.8	0	0.0	-0.1	0.1	0.3	0.1	3.9	8.2	6.7	25.0	3.9	2.5	12.4	7.2	0.6	-6.2	-7.9	-19	-4.8	0	0
2018	0	0	0	0	0	0	0	0	0	0.0	0.1	0.1	0.3	2.6	6.9	5.6	10.4	4.9	1.6	16.5	14.5	8.0	19.3	-6.0	-10	-12	-1.4	0
2019	0	0	0	0	0	0	0	0	0	0.3	0.0	0.9	0.6	4.2	11.7	10.0	16.9	6.2	7.3	15.7	33.9	16.8	34.5	38.5	43.1	19.2	2.2	-0.1
IRR	8%	7%	40%	36%	24%	7%	15%	15%	25%	37%	29%	19%	16%	10%	8%	5%	14%	12%	11%	16%	16%	18%	18%	19%	17%	11%	NA	NA

5.5 Risk and Return Estimates-European PE fund performance

By applying the computed q-factor returns as input in the modified GMM estimator described in chapter 3, the abnormal return and risk exposure of European private equity funds are estimated. The PE fund performance is benchmarked with the q-factor returns which are calculated based on an integrated European company dataset. To avoid overfitting error, a maximum of two factors are incorporated in the estimator. The result of European PE performance is reported in Table 11 and Table 12.

5.5.1 Risk Exposure and Abnormal Return Estimates

As presented in Table 11 Panel A, the average market beta of the buyout fund is 1.32 under specification 1, which is close to the PME beta set (1.3) estimated by Phalippou (2014) for the U.S. buyout funds with a benchmark index of S&P 500. Market beta drops to 0.87 under a CAPM specification accompanying with an abnormal return of 0.3% per month (i.e., 4.1% per annum). The presence of the size factor makes the market beta decrease to a low point of 0.57. The risk exposure of size factor and investment factor are both negative with loadings of -0.77 and -0.34 respectively, suggesting that the return of buyout funds moves in the opposite direction as the low-investment companies and small-sized companies. This is roughly in line with the real practice that buyout funds usually invest more in mature companies. Those target companies generally have stable cash flows with high investment for the purpose of growth, and market capitalization is relatively larger. And Fan et al. (2013) identify a negative size beta in the U.S. buyout sample as well. The ROE factor is positively correlated to the PE returns with a beta coefficient of 0.19. By incorporating the investment factor and ROE factor, the beta of the market factor lies at 1.15 and 1.07 respectively, implying that the European buyout investment has a slightly higher risk exposure to the market compared to the European public securities. This risk coefficient estimation is apparently lower than the one reported by Buchner (2014). He obtains an average market beta of around 2.8 based on an equity sample that excludes the U.K. stocks. But this empirical result of risk exposure is reasonably consistent with some other prior studies (e.g., Ang et al., 2018; Driessen et al., 2012; Fan et al., 2013; Farrelly and Stevenson, 2019; Franzoni et al., 2012;) where beta ranges from 0.85 to 1.5 based on international or U.S. samples.

In terms of venture capital, market beta is estimated at less than 1 (0.97) under specification1 as shown in Panel B. Fan et al. (2013) report an even lower market beta of 0.75 for this asset class. With the exception of this specification, venture capital delivers much higher market betas than the buyout segment at 1.51, 1.93, 2.33 under specification 2 to 4. This is because the inherent high risk of venture capital requires a higher compensation in market return than buyout investments. In a CAPM model, the market beta is 1.6 along with an alpha of -0.5% (-5.6% per annum). On the contrary of buyout funds, the venture capital return is positively related to both size effect and investment effect with the respective beta at 0.56 and 1.89, and the risk coefficient of ROE stands at negative 1.14. These betas demonstrate a stronger co-movement with small-sized companies, low-investment companies, and low-profitability companies. Intuitively, venture capital mainly targets startups in the early stage before the potential explosive growth. These companies normally share the common characteristics of lower market capitalization, less investment on fixed assets, and a temporary low ROE indicator.

The market risk exposure of European venture capital in this research is close to the estimation of Ang et al. (2018), but comparably lower than other studies which report higher average beta coefficient between 2.6 to 2.8 (Korteweg and Sorensen, 2010; Driessen et al., 2012). Concerning these studies use international or US-based samples, a possible explanation for the different estimation is the sample selection bias. Some prior research (Buchner, 2014; Driessen et al., 2012) confirms that venture investments in continental Europe show markedly lower market risk when compared to U.S. counterparts.

With respect to the alpha coefficient, a negative value indicates underperformance and a positive value means outperformance benchmarking with the public index. Venture capital generates a negative alpha of -5.6%, which is significantly lower than that of buyout funds. European venture capital funds underperform the integrated European index by 5.6% while the European buyout funds outperform the integrated European index by 4.1%. As such, buyout funds have better performance than venture capital funds in Europe. This result is close to the research of Fan et al. (2013) which estimates a negative alpha of -4.8% per annum for venture capital and 5.6% for buyout asset class when benchmarking with public index. However, there is a lack of conclusiveness in terms of the abnormal return estimates in the existing literature. The alpha estimated by Farrelly and Stevenson (2019) is -13.4% for opportunity funds and -10.28% for value-added funds under a specification of the Fama-French 3-factor model and a liquidity factor. Driessen et al. (2012) find a slightly negative abnormal performance of buyout funds and an alpha of -12% with the CAPM. Phalippou and Gottschalg (2009) also compute a negative yearly alpha. Most researchers obtain significant positive abnormal returns with a range of 1.9% to 9.3% for PE funds (e.g., Anson, 2013; Buchner, 2014; Franzoni et al.,2012; Kaplan and Schoar, 2005), while some other studies (Cochrane, 2005; Korteweg and Sorensen, 2010) find large alphas that are over 30% per annum.

Table 11. Risk Exposure and Abnormal Return Estimates

This table shows the average monthly abnormal return (α) and risk exposure (β) of European Private Equity Funds using either a one-factor model (specification 1 and 5) or a two-factor model (specification 2, 3, 4). Observations are the FoF portfolios constructed based on the investment starting year. Explanatory variables are the market factor, the size factor, the investment factor, and the profitability factor from the q factor model. β_MKT is the exposure to the market factor. β_ME is the exposure to the size factor. β_I/A is the exposure to the investment factor, and β_ROE is the exposure to the profitability factor.

Panel A: Buyout Funds

	Specification 1	Specification 2	Specification 3	Specification 4	Specification 5
α					0.3%
β_MKT	1.32	0.57	1.15	1.07	0.87
β_ME		-0.77			
β_I/A			-0.34		
β_ROE				0.19	
Pricing Error	9971.54	9918.15	9963.70	9958.11	9907.40

Panel B: Venture Capital Funds

	Specification 1	Specification 2	Specification 3	Specification 4	Specification 5
α					-0.5%
β_MKT	0.97	1.51	1.93	2.33	1.60
β_ME		0.56			
β_I/A			1.89		
β_ROE				-1.14	
Pricing Error	599.08	596.77	591.19	586.58	594.52

5.5.2 Risk Premium and Realized Return

The risk premiums and realized returns for the European private equity funds are calculated and given in Table 12. All the values reported in percentage are annualized returns. Panel A presents the performance result of buyout funds and Panel B presents the results for venture capital funds.

For buyout investment, the market factor generates a return of 11.41% with specification 1. The premium on the size factor is 5.82%, bringing down the market return to 4.92%. With specification 3 and specification 4, premiums on the investment factor and the profitability factor are 5.82%, 1.61%, and the corresponding market returns are 9.92% and 9.28%. According to specification 5, buyout funds have an abnormal return of 4.14% and a market return of 7.5%. From specification 1 to 4, the buyout fund segment earns an excess return of around 11% per annum. Adding the risk-free return, the realized return of the buyout funds reaches approximately 16%. The average pricing error of these 5 specifications amounts to around 9950 and specification 1 has the highest pricing error. The inclusion of the q-factors enhances the CAPM model, to some extent, in the explanatory power of buyout fund performance. With the lowest pricing error (9907), the CAPM model along with an alpha (i.e., specification 5) best specifies the return of the buyout segment.

In Panel B, venture capital has the lowest annual market return at 8.4% under specification 1 where the pricing error is the highest at around 600. The size factor, investment factor, and profitability factor generate negative returns of -4.25%, -9.03%, and -11.34% respectively. Corresponding market returns are 13%, 17% and 20% under specification 2 to 4. The total risk premium is lowest at 7.68% under specification 3 and highest at 13.78% under specification 5. Among the 5 specifications, the one incorporating the ROE factor best specifies the return of venture capital fund. Both specification 3 and specification 4 which include a q-factor decrease the pricing error of the CAPM model.

Apart from the alpha estimation, the realized return also shows that buyout funds outperform venture capitals in Europe. Buyout funds have an average realized return of 16% while venture capital funds have a moderately lower average return at around 13%. This result is aligned with the indication of TVPI, DPI, and IRR measures. With larger market betas, venture capital funds are more sensitively exposed to the market risk than buyout funds. This implies that the extra risk of venture capital does not pay back with an extra return. Besides, the negative abnormal return of 5.6% put venture investment in an even worse situation of poor performance. Venture capitals in Europe have long been asserted to have historically below required returns (Hege et al., 2009). Driessen et al. (2012) suggest that there is too much money perusing too few opportunities, and the assets acquired by venture capital funds are overpriced due to the underestimation of systematic risk. Moreover, in Europe, venture capital funds are sometimes used as tools by the government or associations to support young companies and stimulate the economy. Large amounts of capital for venture capital investments are injected by insurance companies or government-related institutions.

Table 12. Risk Premium and Realized Return of European PE Funds

This table presents the abnormal return, risk premium components, and the cost of capital of European Private Equity Funds with either a one-factor specification (specification 1 and 5) or a multi-factor model (specification 2, 3, 4). Explanatory variables are the market factor, the size factor, the investment factor, and the profitability factor from the q factor model. Observations are the FoF portfolio constructed based on the investment starting year. The risk-free rate R_F and the q-factor returns (R_{MKT}, R_{ME}, $R_{I/A}$, and R_{ROE}) are the annualized average monthly return over the period from January 1981 to December 2019. The abnormal return is also annualized in a compounding way: (1+ Monthly Rate)^12-1. The cost of capital is the sum of all the risk premiums and the risk-free rate. The risk premium is calculated by the q-factor returns times corresponding risk loadings of the PE funds in Table 11. Realized return is the Sum of abnormal return, total risk premium, and risk-free return. Pricing error is calculated with equation 13. The smaller the pricing error, the better the specification explains the return and risk of PE funds. Panel A reports the results for buyout funds and Panel B reports the results for venture capital.

Panel A: Buyout Funds

	Specification 1	Specification 2	Specification 3	Specification 4	Specification 5
α					4,14%
$\beta_{MKT} \times R_{MKT}$	11.41%	4.92%	9.92%	9.28%	7.50%
$\beta_{ME} \times R_{ME}$		5.82%			
$\beta_{I/A} \times R_{I/A}$			1.61%		
$\beta_{ROE} \times R_{ROE}$				1.89%	
Total Risk Premium	11.41%	10.75%	11.52%	11.17%	7.50%
R_F	4.63%	4.63%	4.63%	4.63%	4.63%
Cost of Capital	16.04%	15.38%	16.15%	15.80%	12.13%
Realized Return	16.04%	15.38%	16.15%	15.80%	16.27%
Pricing Error	9971.54	9918.15	9963.70	9958.11	9907.40

Panel B: Venture Capital Funds

	Specification 1	Specification 2	Specification 3	Specification 4	Specification 5
α					-5,57%
$\beta_{MKT} \times R_{MKT}$	8.40%	13.01%	16.71%	20.12%	13.78%
$\beta_{ME} \times R_{ME}$		-4.25%			
$\beta_{I/A} \times R_{I/A}$			-9.03%		
$\beta_{ROE} \times R_{ROE}$				-11.34%	
Total Risk Premium	8.40%	8.76%	7.68%	8.78%	13.78%
R_F	4.63%	4.63%	4.63%	4.63%	4.63%
Cost of Capital	13.03%	13.39%	12.31%	13.41%	18.41%
Realized Return	13.03%	13.39%	12.31%	13.41%	12.83%
Pricing Error	599.08	596.77	591.19	586.58	594.52

6. Conclusion

This study reviews the major modern asset pricing models and discusses the associated comparative studies. The new q-factor model, as one of the most outstanding asset pricing tools, has been rarely tested and applied empirically in non-U.S. regions. With a bottom-up portfolio construction solution, this thesis computes the European q-factor returns based on a comprehensive equity sample and applies these risk factors to benchmarking European private equity fund performance.

The first research question is answered with the below conclusions: **1)** Contrary to some other literature (e.g., Fama and French, 2015; Hou et al., 2015), the empirical result of this study demonstrates a positive correlation between i) return and market capitalization, and ii) return and investment, i.e. stocks of big companies earn higher returns than stocks of small companies and stocks of high-investment companies earn higher returns than stocks of low-investment companies. And without controversy, return positively comoves with the profitability, i.e. stocks of high-profitability companies earn higher returns than stocks of low-profitability companies. **2)** The average monthly premium on the market factor is 0.69%. The size factor (ME) and the investment factor (I/A) generate negative average returns of -0.65% and -0.41% respectively. And the profitability factor (ROE) earns a relatively high average return at 0.79% per month. All the q-factors are significantly different from zero in the European stock market. The European market delivers higher market premium and ROE premium than the U.S. market. **3)** The market factor has the highest volatility over time among all the q-factors. It fluctuates with the megatrend of the equity market and drops to the bottom each time the financial crisis or recession wrecks up the economy. The annual return on ME and I/A remain below zero most of the time from 1981 to 2019. The investment factor premium lags the market factor by one quarter to one year. **4)** In the cross-sectional regression testing, the market beta is significant and positive at around 1 for all the 18 testing portfolios. The size coefficients of small portfolios are more statistically significant and much larger than those of big portfolios, indicating that small portfolios are more sensitive to the size factor. The beta coefficients of the investment factor and the profitability factor are significant for most of the portfolios. Low-investment portfolios are positively exposed to the I/A factor with an average coefficient of 0.53 while high-investment portfolios are negatively exposed to the I/A factor with an average coefficient of -0.47. Factor loadings are positive for all high-profitability stock samples (0.40 on average) and are negative for all low-profitability stock samples (-0.61 on average). The average intercept is reasonably small at 0.0007 and the adjusted R-squares are over 80%, implying that the q-factor model is well specified and has great explanatory power over the return of the European equity market.

For the second research question, this thesis adopts the NPV-based framework of Driessen et al. (2012) as a bridge to connect the q-factor returns with PE fund performance analysis. The master data and historical transactions of European private equity funds are retrieved from Preqin. The testing dataset includes non-liquidated funds and residual NAVs are treated as the cash inflows of the last sample period. Buyout funds and venture capital funds are grouped separately into FoFs based on the vintage year. This thesis ameliorates the NPV-based framework according to the practice of Tausch (2020) by employing an average NPV. The computed q-factor returns and aggregated cash flows of FoFs are embedded in the modified estimator framework. Finally, systematic risk and abnormal returns of European private equity funds are estimated.

The empirical results of European PE fund performance benchmarking can be summarized as below: **1)** Buyout funds have an average abnormal return of 4.1% annually and market beta is around 1.0 on average. Under the 2-factor model specifications, the size effect, I/A factor, and ROE factor contribute an average annual return of 5.82%, 1.61% 1.89%, respectively. Overall, buyout funds slightly outperform the integrated European public security index with an average realized return of 16% per annum. **2)** Venture capital underperforms the integrated European public index by -5.6% per annum. The beta on market factor ranges from 0.97 to 2.33, implying higher market risk than buyout funds. The size factor, investment factor, and profitability factor earn negative average annual premiums at -4.25%, -9.03%, and -11.34% respectively under the 2-factor model specifications. Annual realized return of venture investments amounts to approximately 13%. Despite the higher exposure to market risk, European venture capital funds yield lower returns than buyout investments. **3)** Additionally, this paper finds that the inclusion of the q-factors in the CAPM model does increase the explanatory power for the private equity fund performance. Specification 5 with the alpha and the market factor performs best in explaining the buyout fund return. For venture capital funds, the 2-factor model which contains the market factor and the ROE factor best specifies the return of the venture capital.

This study contributes to the existing literature in several aspects. First, average q-factor returns are computed for the European equity market over a long testing period from 1981 to 2019 based on a large stock sample which covers 34 European countries. Second, this thesis reveals the trend of historical monthly and yearly factor premiums as well as the return patterns in Europe. Third, this research extends the Driessen NAV-based model with the q-factors. It is the first study to analyze the European private equity fund performance with the q-factor model.

Due to the data source limitation, the testing sample of this study does not include most of the delisted companies which could lead to survivorship bias on the empirical results. Besides, considering the high complexity, this thesis abandons the fifth q-factor (expected growth factor) which is computed by regressing the logarithm of Tobin's q, the operating cash flow, and the change in ROE. Hence, further study can be conducted to include the Eg factor and use a full dataset that contains more dead companies. Another direction for the research is to find out the most relevant pricing factors across Europe and calculate the returns for PE benchmarking. Because each market is segmented with its own unique pricing anomalies or the known factors might play a different role and have a different impact on the expected returns in other markets. As Huber and Preissler (2020) pointed out, q-factor models dominate the other pricing models in North America, but are not the winner in Europe.

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