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The Impacts of Ageing Population on Savings and Pension Schemes in a World with Uncertain Life Expectancy

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Abstract:

In the model world with overlapping generations and uncertain life expectancy individuals can live for a maximum of two periods. Only one part of each generation survives the working phase and lives to enjoy retirement. The other part dies after the first half of life. Within this framework it is possible to examine the influences of an ageing population on individual saving behaviour as well as social security systems.

This article analyses the determination of individual savings and per-capita consumption by presupposing constant wage and interest rates. We continue to examine the influence of different pension schemes on individual saving behaviour. It is shown that the introduction of a pension scheme leads to drop in overall per-capita savings regardless of what type of pension scheme - pay-as-you-go or fully funded. Furthermore the influences of a demographic change in the form of increasing life expectancy on individual saving behaviour and pension schemes are examined.

I. Introduction

One possible explanation for the observable phenomenon that each generation passes on property to the next is that individuals are altruistic where their heirs are concerned. Abel[1], on the other hand, suggests that a considerable number of inheritances are in fact "accidental bequests". In a world characterised by uncertain life expectancy, individuals do not know when they will die. If they do in fact die sooner than they expect to, their bequests to their heirs may be considered unintentional.

If, however, individuals live longer than expected, there is a danger that in old-age they will be unable to maintain their desired standard of living owing to a lack of financial means. Motivated by "precautionary saving", these individuals feel compelled to save so as to ensure that their customary level of consumption does not drop if they should happen to live longer than expected. Precautionary saving implies that, at the time of their death, these individuals will still have a certain amount of wealth that their heirs will receive in the form of an *accidental* bequest.

Within the framework of a model with uncertain life expectancy it is possible to examine the effects of an ageing population on existing social security systems. After a study of the World Bank the ageing process of the population in all parts of the world is the greatest challenge for the social security systems in the new century[2].

The effects of uncertain life expectancy on individuals' consumption and savings patterns were first analysed by Yaari [1965]. Since then, Yaari's model has been modified and extended in different directions by a variety of authors.[3]

This article will analyse the determination of individual savings and per-capita consumption by presupposing constant wage and interest rates. We continue to examine the influence of different pension schemes on individual saving behaviour. It will be shown that the introduction of a pension scheme leads to drop in overall per-capita savings regardless of what type of pension scheme - pay-as-you-go or fully funded. Finally, we will analyse the effects of a demographic change in the form of increasing life expectancy on an existing pension scheme.

The present article is based on the work of Abel [1985]. However, whereas Abel analysed solely the effect of fully funded pension insurance ("actuarially fair social security") on consumption and savings, he did not treat pay-as-you-go pension insurance nor the effects of demographic trends such as rising life expectancy.

II. Framework for the model

In the following we shall use the model of overlapping generations developed by Diamond [1965] and extended by Sheshinski and Weiss [1981], Abel [1985, 1987] as well as Eckstein, Eichenbaum, Peled [1985] and Bräuninger [1998].

In a world characterised by uncertain life expectancy individuals live for a maximum of two periods. It is assumed that all individuals survive until the end of the first half of life, i.e. the working phase. At this point in time each individual receives 1+n children. A certain percentage of the parents (1 - p) dies at this time, and the remainder p survives

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[1] See Abel (1985), p. 777.
[2] See World Bank (1994).
[3] See Hakansson (1969), Barro and Friedman (1977), Kotlikoff and Spivak (1981).

until the end of the second half of life, dying after having completed two full life periods. Thus, p describes the probability that an individual will live to experience the second life period. It is assumed that the individuals are aware of the survival probability p. After pensionable age has been reached, life expectancy is no longer uncertain: all survivors will live for the full second period and then die. If N_t represents the number of members of generation t, then, at point in time t+1, there will be (1+n) N_t wage earners and p N_t pensioners.

Since this model presupposes that individuals are selfish, the level of utility of a representative member of generation t depends on that member's consumption during youth and old age, bearing in mind that the level of consumption in old age is not certain owing to the uncertain life expectancy premised in the model. The probability that the individuals will die at the end of the working phase, and thus not be able to consume during their old age, is expressed as 1 - p. The savings amassed by the deceased during the working phase are inherited accidentally, as it were, by their children. If the deceased were still alive, they would enjoy their second life period in full and consume their savings completely. Since the individuals are fundamentally selfish as regards their children, they do not attach any importance to an accidental bequest; an accidental bequest is thus not an argument in their utility function.

The level of utility of the selfish individuals in a world characterised by uncertain life expectancy can be modelled thus:

$$U_t = U(c_{1,t}, p\, c_{2,t+1}), \tag{1}$$

where $c_{1,t}$ and $c_{2,t+1}$ represent consumption in youth and old age respectively. Consumption in old age is included in the utility function with a weighting of p only, the reason being that the individuals' consumption in old age is not certain, but only probable p[4].

In the utility function formulated in (1), the individuals can make two possible mistakes as regards their savings decisions. On the one hand, the savings they amass may be insufficient to finance their consumption in old age should they live for an unexpectedly long time. On the other, the individuals may die sooner than they expect: thus they cannot consume the savings they have amassed and draw no benefit from the compulsory bequest of these savings to their heirs. Had the individuals known in advance that they would die early, they would not have amassed savings but would have spent their entire earnings on consumption in youth so as to enhance their own level of utility.

Whether mistakes were made and, if so, which ones is not clear until the end of the first life period, at which time these mistakes can no longer be corrected. If the individuals die early, they run the risk of making an unplanned bequest from which they themselves derive no benefit. If the save too little, they risk having to reduce their old-age consumption. Since the individuals are aware of this dilemma, they weight their consumption in old age with the appropriate probability in their intertemporal utility function.

III. Optimum amount of savings without pension schemes

In order to determine the optimum savings amount given uncertain life expectancy the intertemporal utility function (1) is specified as follows[5]:

[4] See Abel (1985), p. 779.

$$U_t = \ln c_{1,t} + \frac{p}{1+\delta} \ln c_{2,t+1} \quad \text{for } 0 < p < 1 \text{ and } \delta > 0, \tag{2}$$

where p is the probability of survival and δ the rate of time preference[6]. The utility function (2) thus expresses the expected utility of a representative member of generation t[7]. Furthermore, the utility function (2) assumes that the individuals display a preference for the present, i.e. present consumption of a commodity has a higher utility for them than does future consumption of the same commodity. Owing to this preference for the present, consumption in old age is additionally discounted at the rate of time preference δ.

It is further assumed that, so long as they are young, the individuals will offer, completely unelastically, a unit of their labour and receive in return a fixed working income w. An additional source of income for these young individuals may be an inheritance from their parents. This inheritance e is equivalent to nil if the parents have lived for two full periods and greater than nil if the parents die after the first life period. The available income (= wages + possible inheritance) is used to finance consumption during youth and to build up the desired savings. Thus the following applies for the budget constraint of a member of generation t during the first life period:

$$w_t + e_t = c_{1,t} + s_t. \tag{3}$$

Should the individual reach the second life period, that individual will be in retirement and therefore have no income from employment. Expenditure on consumption in old age then has to be financed by using up the savings amassed in younger years plus interest. Since, as the model assumes, the individuals behave selfishly towards their children and, once they have reached retirement age, their life expectancy is no longer uncertain, they will be tempted to use up their entire assets before they die and to bequeath their children nothing. Thus, the following budget constraint applies in the second life period for the survivors of generation t:

$$c_{2,t+1} = s_t (1 + r_{t+1}). \tag{4}$$

By combining budget constraints (3) and (4), we arrive at the intertemporal budget equation:

$$c_{1,t} + \frac{1}{1+r_{t+1}} c_{2,t+1} = w_t + e_t. \tag{5}$$

Equation (5) states that the present value of expenses must equal the present value of income.

Taking into account the intertemporal budget equation (5), the individuals maximise their level of utility in accordance with (2). The corresponding Lagrange function leads to the optimum condition:

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[5] The advantage of specifying the utility function in this way is that, in contrast to Abel's model (1985), the per-capita variables (consumption, savings and inheritance) can be specified via the individuals' risk attitude without recourse to further assumptions. See Abel (1985), p. 780 and Abel (1987), p. 145f.

[6] See Abel (1985, 1989).

[7] A certain percentage (1 - p) of generation t does not survive the first life period, with the result that their utility from consumption in old age is nil. The remainder p reaches retirement and is able to consume in old age. The expected utility of a representative member of generation t is thus $E(U_t(c_{2,t+1})) = p\, U_t(c_{2,t+1})$.

(6) $c_{2,t+1}/c_{1,t} = \frac{p(1+r_{t+1})}{1+\delta}$.

Condition (6) states that the ratio of consumption in old age to consumption in youth must correspond to the real price ratio. Consumption in old age rises relative to consumption in youth if

- the probability of survival p and the interest rate r_{t+1} rise or
- if the rate of time preference δ falls.

If the values for the probability of survival p are higher or those for the rate of time preference δ lower, more weight is placed on consumption in old age in accordance with the utility function of equation (2). Consequently, utility-maximising individuals will want to increase their consumption in old age in relative terms and forego part of their consumption in youth.

With the same amount of savings and rising interest rates the individuals will have more income in old age since their interest earnings are higher; they can thus finance greater consumption in old age. The quotient $c_{2,t+1}/c_{1,}$ too, rises accordingly.

By combining budget equation (5) with condition (6), we can derive the utility-optimum consumption for both life periods:

(7) $c_{1,t} = \frac{1+\delta}{1+\delta+p}(w_t + e_t)$

(8) $c_{2,t+1} = \frac{p(1+r_{t+1})}{1+\delta+p}(w_t + e_t)$.

Consumption in both life periods thus rises proportionally to the wage rate and any inheritance received. From (4) and (8) it follows for the optimum savings amount:

(9) $s_t = \frac{p}{1+\delta+p}(w_t + e_t)$

$ds_t/dw_t = \frac{p}{1+\delta+p} e_t > 0$

$ds_t/de_t = \frac{p}{1+\delta+p} w_t > 0$.

The derivations of (9) to w_t or e_t are positive, i.e. the formation of savings rises in line with higher values for w_t and e_t. The reason for this is the fact that the individuals' life income increases as the values for w_t and e_t rise. If consumption in old age is a normal commodity, demand for it will increase as income increases (income effect). In order to finance greater consumption in old age, individuals need to save more. As a result, the formation of individual savings will rise.

From (7), (8) and (9) it is clear that both consumption over time and individual savings depend on the size of the inheritance received. However, the size of the inheritance is not the same for all persons. If both parents survive both life periods, the inheritance is nil[8]. The individuals receive an inheritance only if their parents die prematurely. In this case the size of the accidental bequest depends on whether or not the deceased parents themselves received an accidental inheritance from their own parents.

Owing to their differing initial funds we can divide the consumers of generation t into various types. Type 0 comprises those individuals that have received no inheritance, i.e. their parents survived the two life periods. The consumption and savings of Type-0 individuals are defined in the following equations[9]:

$$c_{1,t}{}^{0} = \frac{1+\delta}{1+\delta+p} w_t,$$

$$c_{2,t+1}{}^{0} = \frac{p(1+r_{t+1})}{1+\delta+p} w_t \qquad \text{and}$$

$$s_t{}^{0} = \frac{p}{1+\delta+p} w_t.$$

The consumers of Type 1 are those whose parents died prematurely. Their grandparents, however, survived for two periods and thus made no bequest, i.e. the parents themselves are Type-0 consumers. The size of the inheritance of Type 1 $e_t{}^1$ is determined solely by the savings of the parents (members of generation t-1): these savings earn interest t a rate of r_t and are divided up between 1+n children, i.e.:

$$e_t{}^{1} = \frac{1+r_t}{1+n} s_{t-1}{}^{0}.$$

By substituting this result in (7) to (9), we arrive at the following equations for the consumption and savings of Type-1 consumers:

$$c_{1,t}{}^{1} = \frac{1+\delta}{1+\delta+p} (w_t + \frac{1+r_t}{1+n} s_{t-1}{}^{0}),$$

$$c_{2,t+1}{}^{1} = \frac{p(1+r_{t+1})}{1+\delta+p} (w_t + \frac{1+r_t}{1+n} s_{t-1}{}^{0}) \qquad \text{and}$$

$$s_t{}^{1} = \frac{p}{1+\delta+p} (w_t + \frac{1+r_t}{1+n} s_{t-1}{}^{0}).$$

Type-2 consumers receive an inheritance from their parents, who in turn were Type-1 consumers. The size of the inheritance of a Type-2 consumer is determined as follows

$$e_t{}^{2} = \frac{1+r_t}{1+n} s_{t-1}{}^{1},$$

so that the following apply for the optimum amount of consumption and savings of Type-2 consumers:

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[8] Since the individuals are selfish and a third life period is, by assumption, ruled out, the survivors plan their consumption in old age such that, when they die at the end of the second life period, they will have used up their funds completely.

[9] The consumption and savings plans of Type-0 consumers can be derived from (7) to (9), where $e_t = 0$.

$$c_{1,t}^{2} = \frac{1+\delta}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{1}\right),$$

$$c_{2,t+1}^{2} = \frac{p(1+r_{t+1})}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{1}\right) \qquad \text{and}$$

$$s_t^{2} = \frac{p}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{1}\right).$$

In general, the optimum consumption and savings of Type-i consumers can be determined as follows[10]:

(10) $$c_{1,t}^{i} = \frac{1+\delta}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{i-1}\right),$$

(11) $$c_{2,t+1}^{i} = \frac{p(1+r_{t+1})}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{i-1}\right) \qquad \text{and}$$

(12) $$s_t^{i} = \frac{p}{1+\delta+p}\left(w_t + \frac{1+r_t}{1+n} s_{t-1}^{i-1}\right)$$

for $i = 0,1,...,\infty$ where $s_{t-1}^{-1} = 0$.

In order to determine the average per-capita savings (= the expected value for per-capita savings), we need to know the probability of occurrence of each of the consumer types.

Type 0 occurs if the parents survive the second life period. Thus the probability of occurrence of this type corresponds to the probability of survival: P({"Type-0 consumer"}) = p.

Type 1 occurs if the grandparents survive two periods and the parents die prematurely. Thus the probability of occurrence of this type is: P({"Type-1 consumer"}) = p (1 - p).

Type 2 occurs if the great-grandparents survive two periods and both the grandparents and parents die prematurely. The probability of this occurring is P({"Type-2 consumer"}) = $p\,(1 - p)^2$.

In general, the probability of occurrence of Type-i consumers is: P({"Type-i consumer"}) = $p\,(1 - p)^i$ for $i = 0,1,...,\infty$.

Since $$\sum_{i=0}^{\infty} p\,(1-p)^i = p\,\frac{1}{1-(1-p)} = 1,$$

the completeness of the probability space is proven. Thus the following applies for the average per-capita savings as the expected value of the savings of all consumer types:

[10] See Bräuninger (1998), p. 705.

(13) $$s_t = \sum_{i=0}^{\infty} p\,(1 - p)^i\, s_t^i$$

where $$s_t^0 = \frac{p}{1+\delta+p}\, w_t$$

and $$s_t^i = \frac{p}{1+\delta+p}\,(w_t + \frac{1+r_t}{1+n}\, s_{t-1}^{i-1})$$

$$= s_t^0 + \frac{p}{1+\delta+p}\,\frac{1+r_t}{1+n}\, s_{t-1}^{i-1} \text{ for } i = 1,2,...,\infty.$$

In the following section we shall examine how average per-capita savings change upon introduction of a collective old-age pension scheme.

IV. Optimum per-capita savings in the presence of a pension scheme

Let us assume that the government decides to introduce a pension scheme at the start of period t. A pension level a is fixed as a percentage of income from employment. The pension scheme can either be fully funded or financed on a pay-as-you-go basis.

Fully funded pension scheme

As is the nature of a funding principle, the contributions paid during the period t are invested at the market interest rate and paid out in the period t+1 to the survivors of generation t, so that the following budget equation holds true for the pension scheme:

$$b_{t,K}\, w_t\, (1 + r_{t+1}) = a\, p\, w_{t+1},$$

where a indicates the level of pension in percentage of wage.

From which we can derive the required contribution rate:

$$b_{t,K} = a\, \frac{w_{t+1}}{w_t}\, \frac{p}{1+r_{t+1}}.$$

The introduction of a fully funded pension scheme and the contribution/pension payments it implies mean that it is necessary to modify the budget constraints as per (3) and (4) for both life periods as follows:

(14) $$w_t\,(1 - b_{t,K}) + e_t = c_{1,t} + s_t.$$

(15) $$c_{2,t+1} = s_{t,K}\,(1 + r_{t+1}) + a\, w_{t+1}.$$

From this we can derive the intertemporal budget equation for member of generation t surviving the first life period:

(16) $$c_{1,t} + \frac{1}{1+r_{t+1}}\, c_{2,t+1} = w_t + e_t + (1 - p)\, a\, \frac{w_{t+1}}{1+r_{t+1}}.$$

A comparison of equations (16) and (5) reveals that the lifetime income of persons participating in the fully funded pension scheme is higher. This is due to the fact that the contributions plus accumulated market interest are distributed in full; however, since only a certain number of generation t reaches retirement age, the internal rate of return for them is higher than the market interest rate[11]. However, the higher interest rate is contingent upon those individuals' surviving the first life period[12].

From (16) we can see that the increase in lifetime income is greater,

- the lower the probability of survival and
- the higher the pension level.

A lower probability of survival implies that only a small number of the young generation will survive the working phase. Therefore, the contributions they have paid plus interest earned on those contributions will be shared between a smaller number of survivors, with the result that the latter will profit more from their participation in the public pension scheme. The pension level acts like a lever on the increase in lifetime income.

If we apply the Lagrange function and take into account the modified budget constraints (14) and (15), utility maximisation leads to the familiar optimum condition concerning the distribution of consumption over time:

$$c_{2,t+1}/c_{1,t} = \frac{p(1+r_{t+1})}{1+\delta}.$$

If this is substituted in the modified, intertemporal budget equation (16), we arrive at the following for the utility-optimised consumption in both life periods:

$$(17)\qquad c_{1,t} = \frac{1+\delta}{1+\delta+p}\left[w_t + e_t + (1-p)\, a \frac{w_{t+1}}{1+r_{t+1}}\right],$$

$$(18)\qquad c_{2,t+1} = \frac{p(1+r_{t+1})}{1+\delta+p}\left[w_t + e_t + (1-p)\, a \frac{w_{t+1}}{1+r_{t+1}}\right].$$

A comparison of (17) and (18) with (7) and (8) reveals that the utility-optimised consumption in both life periods rises through the introduction of a fully funded old-age pension scheme. This result is easy to grasp from an economic point of view because, as already explained, by participating in a fully funded pension scheme, the individuals can increase their lifetime income. If consumption is a normal commodity, a higher income means that consumption wishes, too, increase both in youth and old age.

By substituting (18) in (15), we arrive at the utility-optimised formation of savings:

$$(19)\qquad s_{t,K} = \frac{p}{1+\delta+p}(w_t + e_t) - \frac{1+\delta+p^2}{(1+\delta+p)(1+r_{t+1})}\, a\, w_{t+1}.$$

A comparison of (19) and (9) shows that individual savings decrease with the introduction of a fully funded old-age pension scheme since, on the one hand, the collective pension system
partially negates the need to make individual provision for old age and, on the other,

[11] See Abel (1985), p. 782.
[12] See Eckstein, Eichenbaum and Peled (1985), p. 303.

disposable income in youth is diminished by the contributions made to the pension scheme. The drop in individual savings is accompanied by the formation of a per-capita public social fund to the amount of:

$$w_t\, b_{t,K} = a\, w_{t+1} \frac{p}{1 + r_{t+1}}$$

Thus, the overall effect of a fully funded old-age pension scheme on total per-capita savings (= private + public savings) can be expressed as follows[13]:

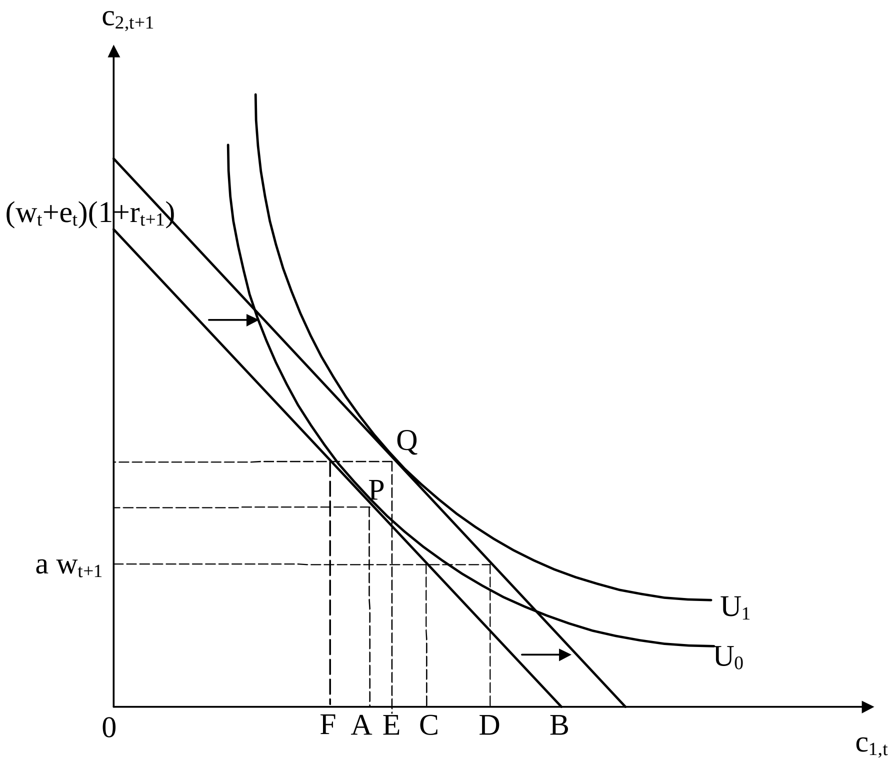
$$a\, w_{t+1} \frac{p}{1+r_{t+1}} - \frac{1+\delta+p^2}{(1+\delta+p)(1+r_{t+1})} a\, w_{t+1} = - \frac{(1-p)(1+\delta)}{(1+\delta+p)(1+r_{t+1})} a\, w_{t+1} < 0$$

In a world characterised by uncertain life expectancy, therefore, the introduction of a fully funded pension scheme reduces total per-capita savings[14]. The higher internal rate of return inherent in a fully funded system means that the pension contributions (= public savings) and private savings are no longer perfectly substitutable commodities[15]. To sum up: *the introduction of a fully funded old-age pension scheme cannot be considered neutral as regards per-capita capital formation and lifetime income*.

Figure 1 shows these connections in graph form. In the absence of a collective pension scheme, individuals would build up private savings to the level shown by the line $\overline{AB}$. The introduction of a fully funded pension scheme increases the individuals' lifetime income. This implies a shift in the budget lines to the right. The individuals reach the higher level of utility U_1 at point Q. In order to finance the higher consumption expenditure in old age connected with point Q, the individuals must build up savings to the amount $\overline{FC}$ in addition to the pension benefits a w_{t+1}. Within the pension system public savings to the amount of $\overline{DB}$ are formed. The positive difference between the lines $\overline{CD}$ - $\overline{FA}$ describes the amount by which total per-capita savings are reduced by the introduction of the fully funded pension scheme.

[13] For p = 1 the effect of the fully funded old-age pension scheme on per-capita savings is nil.
[14] See Abel (1985), p. 783.
[15] See Sheshinski and Weiss (1981), p. 192.

Figure 1 : per-capita savings in a fully funded system



Pay-as-you-go pension scheme

If old-age pensions are financed on a pay-as-you-go basis, the pension contributions of those currently employed are paid out directly to the survivors of the previous generation. The selfish attitude of individuals towards their children means that the members of the initial generation of pensioners will use in full the profit generated by introduction of the scheme to increase their own consumption expenditure in old age.

If the pension level a is fixed by the government, the following budget equation of pension insurance must be fulfilled:

$$b_{t,U}\, w_t\, N_t = a\, w_t\, p\, N_{t-1}$$

From this we can derive the required contribution rate for the pay-as-you-go system:

$$(20) \qquad b_{t,U} = a\, \frac{p}{1+n}\,.$$

Thus, the contribution rate in a pay-as-you-go scheme rises

- if the pension level a increases
- if the probability of survival p increases and
- if the population growth rate n drops.

If a pay-as-you-go scheme is introduced, the budget constraints for both life periods have to be modified as follows:

(21) $w_t (1 - b_{t,U}) + e_t = c_{1,t} + s_t$

(22) $c_{2,t+1} = s_{t,U} (1 + r_{t+1}) + a\, w_{t+1}$.

From these we can derive the intertemporal budget equation:

$$(23) \qquad c_{1,t} + \frac{1}{1+r_{t+1}} c_{2,t+1} = w_t + e_t + a \left(\frac{w_{t+1}}{1+r_{t+1}} - \frac{p w_t}{1+n} \right).$$

Participation in a public old-age pension scheme can either raise or lower lifetime income[16]. If, given constant factor prices, the probability of survival $p < \frac{1+n}{1+r}$, the pay-as-you-go system will improve individuals' income situation. For $n > r$ this will always hold true, since it is assumed that $p \leq 1$.

Applying the Lagrange function, and taking into account the modified budget constraints (21) and (22), optimisation of the utility function (2) leads to the optimum condition:

$$c_{2,t+1}/c_{1,t} = \frac{p(1+r_{t+1})}{1+\delta}.$$

Taking into account the intertemporal budget equation (23), it follows for the utility-optimised consumption in both life periods that:

$$(24) \qquad c_{1,t} = \frac{1+\delta}{1+\delta+p} \left[w_t + e_t + a\left(\frac{w_{t+1}}{1+r_{t+1}} - \frac{p w_t}{1+n} \right) \right],$$

$$(25) \qquad c_{2,t+1} = \frac{p(1+r_{t+1})}{1+\delta+p} \left[w_t + e_t + a\left(\frac{w_{t+1}}{1+r_{t+1}} - \frac{p w_t}{1+n} \right) \right].$$

Compared with the initial situation (i.e. without a collective pension scheme), per-capita consumption in both life periods can rise, fall or remain the same. Per-capita consumption in both life periods rises when $\frac{w_{t+1}}{1+r_{t+1}} > \frac{p w_t}{1+n}$ applies, i.e. if lifetime income is enhanced through participation in the pension scheme .

Taking (25) into account of in (22), we arrive at the optimum per-capita savings amount achievable in the context of a pay-as-you-go pension scheme:

$$(26) \qquad s_{t,U} = \frac{p}{1+\delta+p} (w_t + e_t) - \frac{1+\delta}{(1+\delta+p)(1+r_{t+1})}\, a\, w_{t+1} - \frac{p^2}{(1+\delta+p)(1+n)}\, a\, w_t.$$

Here, too, it holds true that the introduction of a pay-as-you-go pension scheme lowers individual savings, for which there are two reasons: firstly, the existence of a public pension scheme partially negates the need to provide for one's old age and, secondly,

[16] For the initial generation of pensioners participation in the pension scheme will always increase their lifetime income.

because they have to pay pension contributions during their working life, the individuals have less disposable income to invest in saving.

Since no public social fund is amassed in a pay-as-you-go system, overall per-capita savings fall in line with declining private savings. The introduction of a pay-as-you-go pension scheme thus leads to reduction a in overall per-capita savings.

V. Effects of changes in life expectancy

There is an observable demographic trend towards an ever higher percentage of older people in the total population. This fact can be reflected in the present model by an increasing probability of survival p; for a higher probability of survival implies that a larger percentage of workers will survive the first life period and reach retirement age. And this will increase the percentage of older people relative to the entire population.

In the absence of a social security scheme

If no old-age pension scheme is in place, the optimum amount of individual savings in accordance with (9) is:

$$s_t = \frac{p}{1+\delta+p}(w_t + e_t).$$

Derivation of this equation for the probability of survival p results in:

$$(27) \qquad ds_t/dp = \frac{1+\delta}{(1+\delta+p)^2}(w_t + e_t) > 0,$$

i.e. the individuals make greater provision for old age when they realise that their probability of survival is greater. This makes sense because the higher the probability of survival, the greater the "risk" that they will live too long. In other words, the probability is greater that the individual will make a wrong decision, i.e. that they do not amass enough savings to maintain their desired living standard in old age. Consequently they will try to save more to counteract the increased "risk" of longevity.

In accordance with the utility function (2) rising probability of survival means that consumption in old age is weighted higher because there is now a higher probability that the individuals will live to experience a second life period. The individuals utility-maximising approach means that they prefer - in relative terms - greater consumption in old age. In order to finance higher consumption in old age, they must make greater provision for that life period.

With a fully funded social security system

In contrast to a world characterised by certain life expectancy[17], in one marked by uncertain life expectancy the demographic trend towards an ageing population has a direct effect on the financing structure of a fully funded old-age pension scheme. Owing to

$$b_{t,K} = a \frac{w_{t+1}}{w_t} \frac{p}{1+r_{t+1}} \qquad \text{and}$$

[17] See Blanchard and Fischer (1998), p. 111.

$$db_{t,K}/dp = a\ \frac{w_{t+1}}{w_t}\ \frac{1}{1+r_{t+1}} > 0$$

the contribution rate to the pension scheme has to be increased if, for socio-political reasons, a certain pension level $\bar{a}$ has to be maintained. The reason for this is that, as the probability of survival rises, a larger percentage of each generation will survive the first life period and begin drawing a pension. This leads to an overall rise in expenditure on pensions and to an increase in the contribution rate.

Total differentiation of private savings in accordance with (19)

$$s_{t,K} = \frac{p}{1+\delta+p}(w_t + e_t) - \frac{1+\delta+p^2}{(1+\delta+p)(1+r_{t+1})}\ a\ w_{t+1}$$

results in:

$$(28) \qquad ds_{t,K} = \frac{1+\delta}{(1+\delta+p)^2}(w_t + e_t)\ dp + \frac{(1+\delta+p^2)-2p(1+\delta+p)}{(1+\delta+p)^2(1+r_{t+1})}\ a\ w_{t+1}\ dp - \frac{1+\delta+p^2}{(1+\delta+p)(1+r_{t+1})}\ w_{t+1}\ da.$$

If the pension level is kept constant (da = 0), we can derive from (28) the following formula for the change in the formation of private savings:

$$(29) \qquad ds_{t,K} = \frac{1+\delta}{(1+\delta+p)^2}(w_t + e_t)\ dp + \frac{1}{(1+r_{t+1})}\ a\ w_{t+1}\ dp - \frac{2p}{(1+\delta+p)(1+r_{t+1})}\ a\ w_{t+1}\ dp.$$

This type of adjustment produces contradictory effects on private savings. On the one hand, the individuals with a fixed working income will save more in order to counteract the increased "risk" of longevity. In the utility function, consumption in old age is weighted higher as the probability of survival rises, with the result that the individuals will desire correspondingly greater consumption in old age. They will have to make greater provision to achieve this. This has a positive effect on the formation of private savings.

On the other hand, the necessary increase in the contribution rate for the old-age pension means that individuals have less disposable income during the working phase and can thus save less. This has a negative effect on private savings.

Since the pension insurance contribution is higher, the per-capita public social fund is also higher. In Section IV we identified the expression

$$\frac{(1-p)(1+\delta)}{(1+\delta+p)(1+r_{t+1})}\ a\ w_{t+1}$$

as being the amount by which total per-capita savings (= private + public savings) are lowered by the introduction of a fully funded pension scheme. Derivation for p results in:

$$- \frac{(2+\delta)(1+\delta)}{(1+\delta+p)^2(1+r_{t+1})} a\, w_{t+1} < 0,$$

i.e. the amount by which total per-capita savings fall diminishes as the probability of survival p rises. The reason for this is that, as the probability of survival rises, the internal rate of return of the fully funded system falls. Thus, the reduction in the formation of private savings triggered by the introduction of a fully funded pension is not so strong and total per-capita savings fall to a lesser extent. To turn the argument on its head: the higher the probability of survival, the higher the per-capita savings.

If the rise in the probability of survival occurs unexpectedly, it is not possible in the short term to maintain a set pension level. If the members of generation t do not recognise until the period t+1 that far too many of their generation have survived the working phase, it is not possible in retrospect to increase the contribution rate so as to maintain the existing pension level. The only option they have left is to distribute the public pension assets (= contributions of members of generation t + interest) among an unexpectedly high number of pensioners. As a result, the pension level is lowered while the contribution rate remains constant. The required drop in the pension level can be calculated via total differentiation of the contribution rate formula:

$$db_{t,K} = \frac{w_{t+1}}{w_t} \left[\frac{p}{1+r_{t+1}}\, da + \frac{a}{1+r_{t+1}}\, dp \right].$$

Taking into account the constant contribution rate ($db_{t,K} = 0$), we can deduce the following for the required pension adjustment:

$$da = - \frac{a}{p}\, dp.$$

If we substitute this in equation (28), we arrive at the change in the formation of private savings:

$$ds_{t,K}/dp = \frac{1+\delta}{(1+\delta+p)^2} (w_t+e_t) + \frac{(1+\delta)\left[p(1-p)+(1+\delta+p)\right]}{(1+\delta+p)^2(1+r_{t+1})p} a\, w_{t+1} > 0.$$

If the pension level is lowered and the contribution rate left unchanged, individual savings rise for two reasons.

Once individuals grasp that their "risk" of longevity rises in line with a higher probability of survival, they will try and save more so that they can maintain their desired standard of living in old age despite living longer. However, the individuals expect that, with an ageing population, the pension level, and thus the internal rate of return on their contributions to pension insurance, will fall: what they pay into the system will result in lower pension benefits. Consequently, the individuals will save more in order to offset the expected drop in their pensions.

Since the per-capita public social fund will not change if contribution rates are constant and individual savings, as explained above, will rise, total per-capita savings (pubic + private savings) will rise in line with a higher probability of survival.

To sum up: if a fully funded old-age pension scheme is in place, total per-capita savings will rise in line with a higher probability of survival *regardless* of what type of adjustment is made.

With a pay-as-you-go social security system

Since there are no public savings in a pay-as-you-go pension scheme, total per-capita savings are identical with private savings. In a pay-as-you-go pension scheme, too, demographic trends like an ageing population have a direct effect on the financing structure. Either the pension level must be lowered and the contribution rate left unchanged, or the latter must be increased to keep the former constant. Although, of course, any combination of the two measures is possible, every adjustment must satisfy the following condition:

$$db_{t,U} = \frac{p}{1+n}\, da + \frac{a}{1+n}\, dp.$$

Total differentiation of individual savings in accordance with (26) results in:

$$(30) \quad ds_{t,U} = \frac{1+\delta}{(1+\delta+p)^2}\,(w_t + e_t)\, dp + \frac{1+\delta}{(1+\delta+p)^2(1+r_{t+1})}\, a\, w_{t+1}\, dp -$$

$$- \frac{p(2+2\delta+p)}{(1+\delta+p)^2(1+n)}\, a\, w_t\, dp - \frac{1+\delta}{(1+\delta+p)(1+r_{t+1})}\, w_{t+1}\, da - \frac{p^2}{(1+\delta+p)(1+n)}\, w_t\, da.$$

If the pension level is kept constant and the contribution rate lowered, the following applies to the change in individual savings for $da = 0$:

$$(31) \quad ds_{t,U} = \frac{1+\delta}{(1+\delta+p)^2}\,(w_t + e_t)\, dp + \frac{1+\delta}{(1+\delta+p)^2(1+r_{t+1})}\, a\, w_{t+1}\, dp -$$

$$- \frac{p(2+2\delta+p)}{(1+\delta+p)^2(1+n)}\, a\, w_t\, dp.$$

An increase in the contribution rate owing to a higher probability of survival has two contradictory effects on the formation of private savings.

If the probability of survival rises, consumption in old age will be weighted higher in the intertemporal utility function (2); as a result, individuals will be compelled to increase both their planned consumption in old age and, thus, the amount saved in younger years (substitution effect). This has a positive effect on the formation of savings.

On the other hand, the higher contribution rate means that the individuals have a lower disposable income during their working phase, thus limiting the amount they can save (income effect). This effect is a negative one.

If the pension level is lowered and the contribution rate held constant, the required pension adjustment for $db_{t,U} = 0$ is:

$$da = -\frac{a}{p}\, dp.$$

Taking this adjustment of the pension level into account, we can, after several transformations to reflect the change in savings formation, deduce the following from (30):

$$(32) \qquad ds_{t,U} = \frac{1+\delta}{(1+\delta+p)^2} e_t \, dp + \frac{(1+\delta)(1+\delta+2p)}{(1+\delta+p)^2 (1+r_{t+1})p} a \, w_{t+1} \, dp + \frac{(1+\delta)(1+n-ap)}{(1+\delta+p)^2 (1+n)} w_t \, dp > 0 \text{ for } dp > 0.$$

Thus, individual savings rise with an ageing population. With this type of adjustment savings increase for two reasons. Firstly, individuals tend to save more in order to offset the increased "risk" of longevity triggered by a higher probability of survival (precaution effect). Secondly, owing to the fact that they can expect to receive lower pensions individuals will have to make more private provision for their old age in order to maintain their desired standard of living in old age (income effect).

To sum up: if a pay-as-you-go pension scheme is in place, the effect of a higher probability of survival on per-capita savings *is dependent* on the type of adjustment made in pension insurance. Per-capita savings will rise if the pension level is lowered and the contribution rate kept constant. If the pension level remains constant and the contribution rate is increased, it is unclear what the effect of a higher probability of survival will be on total per-capita savings.

VI. Summary

In the model world examined here, in which life expectancy is uncertain, individuals can live for a maximum of two periods. Only part of each generation survives the working phase and lives to enjoy retirement. However, the individuals themselves do not know whether they will survive to enjoy the second life period, and they must take this uncertainty into account when making their savings decisions. They can make two possible errors in this respect. If they save too little, they will not be able to maintain their desired standard of living. On the other hand, they will derive no utility from their savings if do not reach retirement age. The latter case results in a compulsory bequest, from which the selfish individual does not derive any utility. We also speak of an *accidental* bequest in this context. The uncertainty of life expectancy is taken into account through use of the expected utility.

In contrast to a world characterised by certain life expectancy, the introduction of a fully funded pension scheme into our model world leads to a drop in overall per-capita savings. This violates the neutrality theorem. Since the pension contributions plus interest are paid out to the surviving contributors only, the internal rate of return of the pension scheme is higher than the capital-market interest rate. As a result, private capital formation diminishes to such an extent that the public pension assets amassed cannot offset the drop in private savings.

The introduction of a pay-as-you-go pension scheme, too, reduces per-capita savings because the savings motive is in part satisfied by participation in the public pension scheme. Since no public pension fund is built up in a pay-as-you-go system, the full impact of the drop in private savings is felt in lower overall capital formation.

Unlike in a world with certain life expectancy, demographic change in the form of an ageing population directly triggers an adjustment in the pension scheme, whether the latter is fully funded or pay-as-you-go. The resulting effect on private savings will differ depending on the type of adjustment made. The following hold true:

- A higher probability of survival means that the "risk" of longevity rises. Individuals behave rationally in that they will make greater provisions to reduce this "risk".

- If the pension level is lowered and the contribution rate kept constant, the individuals will endeavour to save more money to offset the expected lower pension. This has a positive effect on individual savings.

- If the pension level is kept constant and the contribution rate increased, this will reduce the individuals' disposable income - and thus the amount they can save - during their working phase. This effect is negative.

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